ILLINOIS STATE BOARD OF EDUCATION

MATHEMATICS HIGH SCHOOL ALGEBRA

Theory of Action: Academic standards represent a collective commitment around what students should learn each year. The state assessment asks students to demonstrate their knowledge, skills, and understanding related to these standards using a common measure. The resulting data allows us to see patterns in performance that should guide school and district improvement, helping identify areas of strength and opportunity.

Role of Performance Level Descriptors in Defining Proficiency: Performance level descriptors bridge the state assessment to classroom instruction and the systems of formative assessments that guide local instruction and choices about individual students. *Academic proficiency represents a range of observable student performance characteristics*. There are multiple pathways to proficiency, and students rely upon their strengths differently within that range of performance.

Proficiency and Difficulty: A student's ability to demonstrate proficiency is influenced by the complexity of the texts or stimuli presented, tasks they're asked to complete, and the contexts in which they are engaged. As student performance improves, students are typically able to handle more challenging texts/stimuli, tasks, and contexts, and are able to demonstrate their skills and knowledge more accurately and consistently.

Algebra	***Student performance indicates the ability to…***			
Claim 1	**Below Proficient**	**Approaching Proficient**	**Proficient**	**Above Proficient**
A.CED.1	Write and evaluate numerical expressions involving whole-number and integer exponents.	Given an equation that models a situation, evaluate the model at a specific input value.	Given a table or verbal description of a linear situation, create an equation or inequality to model the situation, and use the model to solve problems.	Given a table or verbal description of a simple rational or exponential situation, create a one-variable equation or inequality to model the situation. Use the model to solve problems.
	Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	Given an equation that models a situation, create a table of output values given specific input values.	Given a table of input and output values, create an equation that matches the table, including linear, quadratic, and simple exponential (i.e., $y = 3^x$) equations.	Given a table of input and output values, create an equation that matches a simple rational equation (i.e., $y = \frac{2}{x-3}$).
	Write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable.		Use created linear, quadratic, and simple exponential equations to solve problems (i.e., determine what input value will result in a given output value).	Given a table of input and output values, create an equation that matches a more complicated exponential situation (i.e., $y = 3.48(1.0425)^x$.

Claim 1	Below Proficient	Approaching Proficient	Proficient	Above Proficient
A.CED.2 **A.CED.3**	Given an equation or inequality and a possible solution, determine if the solution satisfies the equation or inequality (i.e., plug in the given values for the corresponding variables to verify the solution).	Solve a system of equations using graphs and tables.	Create a system of linear inequalities that serve as constraints for a real-world model.	Create a system of nonlinear inequalities (or a combination of linear and nonlinear inequalities) that serve as constraints for a real-world model.
		Given a graph of an inequality, determine a solution to the relationship.	Create a graph for a system of linear inequalities and solve the system graphically.	Create a graph for a system of nonlinear inequalities (or linear and nonlinear inequalities) and solve the system graphically.
	Identify the relationship between the dependent and independent variables using graphs and tables and relate these to the equation.	Create a graph or a table that represents a relationship between the dependent and independent variables (equality and inequality).	Assess the solution to a linear system of inequalities as viable or non-viable (i.e., does the solution satisfy all constraints).	Create and apply a literal equation; rearrange formulas to highlight a quantity of interest using any necessary operations (i.e., square root, power).
A.CED.4	Use variables to represent quantities in a real-world or mathematical problem and construct equations and inequalities to solve problems by reasoning about the quantities.	Use variables to represent quantities in a real-world or mathematical problem and construct two-variable equations and inequalities to solve problems by reasoning about the quantities.	Use variables to represent quantities in a real-world or mathematical problem and construct multi-variable equations and inequalities to solve problems by reasoning about the quantities.	Use variables to represent quantities in a real-world or mathematical problem and construct multi-variable equations and inequalities to solve problems by reasoning about the quantities.
	Apply a formula when given the necessary input information.	Given two different algebraic forms of a literal equation, identify the algebraic steps needed to show they are equivalent.	Given a literal equation, rearrange formulas to highlight a quantity of interest using addition, subtraction, multiplication, and division. Rewrite an expression or a formula to express one variable in terms of the other(s).	Create and rearrange literal formulas to highlight a quantity of interest using addition, subtraction, multiplication, and division. Rewrite advanced expressions or formulas involving multiple steps to express one variable in terms of the other(s).

Algebra	***Student performance indicates the ability to…***			
Claim 2	**Below Proficient**	**Approaching Proficient**	**Proficient**	**Above Proficient**
A.SSE.1, A.SSE.1a A.SSE.1b	Substitute whole numbers into expressions and evaluate, as needed.	Substitute rational numbers into expressions and evaluate, as needed.	Interpret the structure of expressions. Interpret expressions that represent a quantity in terms of its context. For example, in $A = L \times W,$ a student should be able to determine the change in area if L is increased or decreased.	Rewrite an expression as a set of one or more operations and describe what a change in one will do to the others if all other factors remain constant. (For example, if $x^2y = 1$, how does increasing y change x?)
	Identify parts of a linear expression (coefficient, variable, term).	Identify an expression and distinguish it from an equation.	Interpret parts of quadratic expressions, such as terms, factors, and coefficients, in any form. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.	
A.SSE.2	Subdivide expressions into single-operation parts (e.g., $3x - 5y + 2x - y = (3x + 2x) + (-5y - y)$).	Interpret the structure of a square as having multiple parts. Recognize a simple difference of a quadratic term and a perfect square constant that can be factored ($x^2 - 25$ equivalent to $(x - 5)(x + 5)$).	Recognize and factor the special case difference of two squares when the two terms of the expression can be rewritten as quadratic terms. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.	Recognize and factor the perfect square trinomial with two variables ($(x - y)^2$ as $(x^2 - 2xy + y^2)$). For example, see $x^{12} - y^{12}$ as $(x^4)^3 - (y^4)^3$ and thus recognize it as difference of cubes factored as $(x^4 - y^4)(x^8 + x^4y^4 + y^8)$.
	Factor out a simple GCF ($5(5 - x) = 25 - 5x$).	Recognize and factor a binomial squared with a constant and a variable that has a coefficient $= 1$.	Recognize and factor a binomial squared where one of the variables has a coefficient > 1. Recognize and factor the perfect square trinomial with two variables ($(x - y)^2$ as $(x^2 - 2xy + y^2)$).	
A.SSE.3	Rewrite simple algebraic expressions in one variable using algebraic properties (e.g., combine like terms, use the distributive property).	Identify equivalent expressions given different forms of the relationships between quantities/variables.	Rewrite an algebraic expression purposefully to solve a problem or reveal properties of the expression (e.g., algebraically determine if two equations are for the same line).	Apply different forms of equivalent expressions to multi-step problems where the given form is more suitable for the context of the problem/relationship.

A.SSE.3a	Identify a GCF for a quadratic and use distributive property to rewrite with GCF factored out.	Determine the x intercepts from the factored form. Construct an equation in factored form given the x-intercepts.	Factor a quadratic where $a = 1$ and determine the x intercepts.	Factor a quadratic with $a > 1$ and/or GCF and then determine the x intercepts.
A.SSE.3b	Identify the maximum or minimum value of a function from its graph or table.	Identify the vertex of a parabola from the vertex form. Given an equation for a parabola in standard form with $a = 1$, write an equivalent equation in vertex form.	Solve a quadratic equation by completing the square with the $a = 1$. Identify the vertex and concavity of a quadratic function from the vertex form. Identify the maximum or minimum value of a quadratic function in vertex form. Apply symmetry to determine the axis of symmetry from the factored form.	Solve a quadratic equation by completing the square with the $a \neq 1$.
A.SSE.3c	Recognize the structure of an exponential expression (base versus exponent).	Use properties of integer exponents (addition, subtraction) to manipulate with the base (e.g., rewrite $4 \cdot 2^x$ as 2^{x+2} or vice versa).	Use properties of integer and rational exponents (negative, zero, power to the power) to manipulate with the base in real world context (e.g., changing compound interest to an equivalent value given different time interval).	Create an exponential function from a real-world context and use properties of rational exponents to manipulate with the base and exponent.
A.SSE.4	Find the sum of an explicitly defined geometric series.	Find the sum and/or any missing terms in a geometric series given the final sum or the formula.	Use the formula for a finite geometric series with a common ratio, not 1, to solve problems.	Given a context, derive the formula for the sum of a finite geometric series (when the common ratio is not 1) and use the formula to solve problems. For example, calculate annuity values.

Algebra	*Student performance indicates the ability to…*			
Claim 3	**Below Proficient**	**Approaching Proficient**	**Proficient**	**Above Proficient**
A.APR.1	Perform operations on monomials: add/subtract like terms and multiply a monomial by a polynomial using exponent properties.	Perform addition, subtraction, and multiplication on binomials (e.g., binomial squares).	Perform arithmetic operations on polynomials. Recognize and connect the concept that polynomials, when added, subtracted, and multiplied, will always result in a new polynomial.	Calculate with quantities expressed as polynomial expressions (e.g., find area/perimeter of a quadrilateral which side lengths are polynomials).
Claim 3	**Below Proficient**	**Approaching Proficient**	**Proficient**	**Above Proficient**
A.APR.2 **A.APR.3**	Identify the zeros of a polynomial function given the graph.	Rewrite a polynomial using its zeros as factors. Solve simple factorable polynomials.	Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.	Apply the remainder theorem to a polynomial relationship to find zeros and nonzero values in the domain.
	Determine the zeros given the factored form of a polynomial.	Plot the zeros of polynomial function on a coordinate plane.	Verify zeros of polynomials using the remainder theorem. (e.g., for a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.)	
A.APR.4		Given a polynomial identity and value(s) for the variable(s), use substitution to confirm the identity is true for that value(s).	Determine quadratic polynomial identity using variables only.	Determine higher order polynomial identities using variables.
	Multiply two binomials.	Recognize the result of multiplying two binomials is an identity that is true for all values of the variable.	Establish difference of squares identify by expanding $(a + b)(a - b)$.	Use polynomial identities to describe numerical relationships (sum/difference of cubes, generate Pythagorean triples). $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$
			Use difference of squares to describe numerical relationships or calculate quantities (i.e., calculate $15^2 - 6^2$).	
			Establish square of binomial identity by expanding $(a + b)^2$.	

A.APR.6	Simplify rational expressions with two monomials.	Divide a binomial or trinomial by a monomial. Rewrite $\frac{a(x)}{b(x)}$ where $a(x)$ and $b(x)$ are both linear as $q + \frac{r}{b(x)}$, where q and r are constants, either by inspection or polynomial division.	Use polynomial division to rewrite a rational expression $\frac{a(x)}{b(x)}$ into a form that includes or less and a rational expression with the degree of the remainder (numerator) being less than the divisor (denominator), where $a(x)$ and $b(x)$ are of degree 3 or less.	Use polynomial division to rewrite a rational expression $\frac{a(x)}{b(x)}$ into a form that includes or less and a rational expression with the degree of the remainder (numerator) being less than the divisor (denominator), where $a(x)$ and $b(x)$ are of any degree.

Algebra	***Student performance indicates the ability to…***			
Claim 4	**Below Proficient**	**Approaching Proficient**	**Proficient**	**Above Proficient**
A.REI.1-3	Verify a solution to a linear, quadratic, radical, or rational equation by substituting the given solution for the variable.	Use properties of equality to isolate the variable in a one-step linear, quadratic, or radical equation.	Use properties of equality to isolate the variable in any linear equation or inequality, including equations with variable coefficients.	Solve multi-step rational and multi-step radical equations in one variable.
	Verify a solution to a linear, quadratic, radical, or rational equation by substituting the given solution for the variable.	Use properties of equality to solve a one-step linear inequality.	Use properties of equality to isolate the variable in simple rational and simple radical equations and inequalities. Be sure to identify any extraneous solutions.	Explain how extraneous solutions arise and how limiting the domain impacts the final solution.
A.REI.4	Solve one-step linear equations and inequalities with one variable.	Solve two-step linear equations and inequalities with one variable.	Solve multi-step equations and inequalities with one variable. Solve quadratic equations in one variable.	Solve multi-step, multi-variable equations, and inequalities. Solve multi-variable quadratic equations.
A.REI.4a-4b	Solve one-step quadratic equations and inequalities with one variable (e.g., $x^2 = 25$).	Solve two-step quadratic equations or inequalities in one variable by inspection or taking square roots (e.g., $x^2 + 2 = 27$).	Use completing the square to transform a quadratic equation when $a = 1$ into the $(x - p)^2 = q$ form. Solve quadratic equations in one variable by completing the square.	Use completing the square to transform any quadratic equation into the $a(x - p)^2 = q$ form.
		Solve nicely factorable quadratic equations by factoring.	Solve quadratic equations in one variable using the quadratic formula. Recognize when the quadratic formula yields complex solutions as $a \pm bi$.	Derive the quadratic formula by completing the square.

A.REI-5-6	Given a graph, identify the solution to a system of two linear equations.	Given a linear equation, construct equivalent linear equations, either by solving for a variable (to assist with substitution) or by scaling the equation by a multiplier (to assist with elimination).	Use properties of equality (e.g., substitution and elimination) to solve a system of equations where both equations are linear. Solve a system of two linear equations algebraically and graphically.	Use any method to solve a system of equations that includes any two or three equations, including non-linear cases.

Claim 4	Below Proficient	Approaching Proficient	Proficient	Above Proficient
A.REI.7	Given a graph, identify the solution to a system of one or two nonlinear equations.	Solve a system of equations where one equation is linear and the other is quadratic graphically using technology.	Solve a system of equations where one equation is linear and the other is quadratic or an equation of a circle, both algebraically and graphically.	Use any method to solve a system of equations that includes both linear and nonlinear equations.
A.REI.10-A.REI.12	Given a graph of a single function or inequality, generate a set of ordered pairs represented.	Given a graph of a system of equations or inequalities, identify an ordered pair as a solution to a system.	Solve a system of linear equations or linear inequalities by graphing. Find approximate solutions to a nonlinear system of equations (e.g., using technology to graph the functions). Equations can be polynomial, rational, absolute value, exponential, and/or logarithmic.	Use the solutions to a system of equations to answer questions when the system represents restrictions on the model. Find appropriate solutions for a system using advanced techniques (e.g., successive approximations).