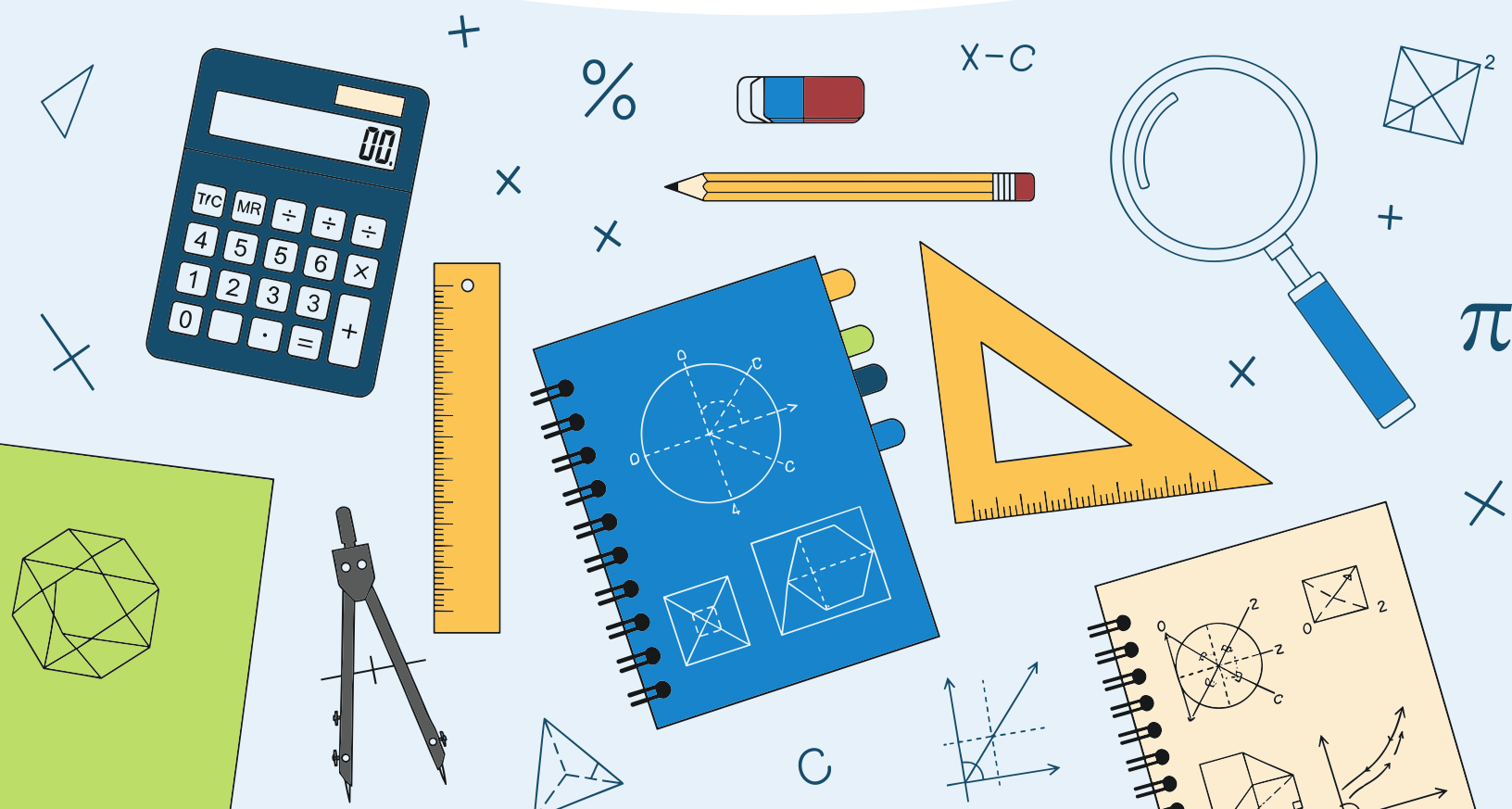




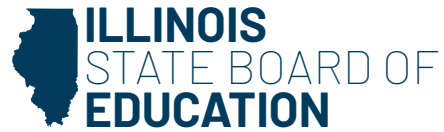
# Illinois Comprehensive Numeracy Plan





# **Illinois Comprehensive Numeracy Plan**

Illinois State Board of Education  
June 2026



**Illinois State Board of Education**

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ISBE appreciates notification where errors occur so they may be corrected in subsequent printings.

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## **Acknowledgements**

The Illinois State Board of Education (ISBE) extends its deepest gratitude to the many educational partners who contributed to the development of the Illinois Comprehensive Numeracy Plan at every stage of its creation.

This process began in April 2025 with a statewide needs assessment. Input gathered from the field helped ISBE better understand challenges, priorities, and areas of support in mathematics instruction. In June 2025 ISBE hosted the Numeracy Summit, which brought together educators, higher education faculty, school leaders, community organizations, nonprofit advocates, and policymakers to establish a shared vision for improving mathematics outcomes in Illinois. Participants explored best practices, engaged in meaningful dialogue around effective instructional strategies, and identified key components of a statewide numeracy plan. During the summit, parents, educators, administrators, mathematics organizations, and other educational partners volunteered to lead the initial drafting of the plan. This work began with the pen in the hands of the stakeholders, whose dedication of time and expertise resulted in the first draft of the Illinois Comprehensive Numeracy Plan. ISBE extends its sincere gratitude to the drafting team for their work in creating the foundation for the plan as well as their steadfast commitment to advancing equitable numeracy instruction throughout the state.

ISBE also recognizes the educational partners who participated in the statewide listening tours, focus groups, and public comment opportunities. The feedback provided was invaluable and played a critical role in shaping and refining the plan throughout its development. The breadth of perspectives and the depth of expertise ensured that the diverse needs of students across all regions of Illinois remained central to this work.

With the completion of the Illinois Comprehensive Numeracy Plan, ISBE looks forward to supporting its implementation as a catalyst for meaningful and sustained improvement in mathematics education. The level of collaboration and dedication demonstrated throughout this process exemplifies the strength of the education community, and ISBE acknowledges educators and educational partners across the state for their unwavering commitment to students. As this work moves forward, ISBE stands as a committed partner alongside educational communities, recognizing that numeracy is not only an academic priority but an essential skill that empowers students to navigate and contribute to the world around them.

Together, this work represents a shared commitment to shaping a stronger future for mathematics education in Illinois.

Thank you,

The ISBE Numeracy Team

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# Illinois Comprehensive Numeracy Plan Overview

## Vision and Purpose

Articulates the Numeracy Plan's goals and serves as a call to action.



## Numeracy Skills

Students will build numeracy skills through tasks that develop reasoning, fluency, and real-world problem solving.



## Professional Learning

Educators will use evidence-based numeracy strategies to strengthen students' mathematical understanding and confidence.



## Leadership

Leaders will promote a culture of numeracy through targeted professional learning and data-informed instructional support.



## Workbook Pages

The workbook pages that accompany Sections 1-3 are intended to foster reflection and support meaningful dialogue among educational partners.



## Tools and Resources

A list of tools and resources designed to support districts, educators, and families.

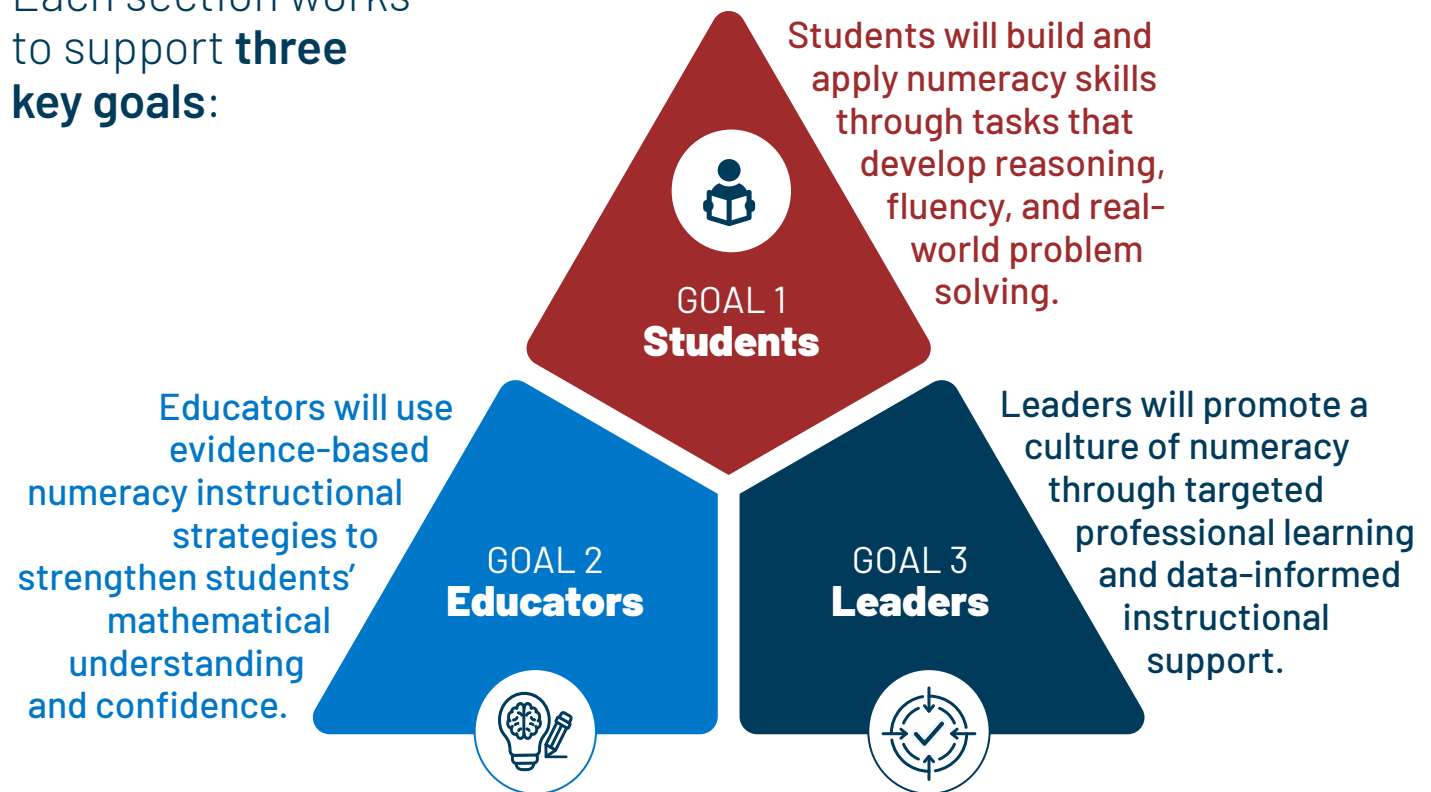


## VISION AND PURPOSE

### ISBE's Commitment to Equitable Numeracy Instruction

The Illinois Comprehensive Numeracy Plan (ICNP) is founded on the principle that every student, in every classroom, deserves instruction that empowers them to be confident and capable mathematical thinkers. The Illinois State Board of Education is committed to ensuring equitable access to high-quality numeracy instruction for all students, including culturally, linguistically, and academically diverse student populations. This commitment requires intentionally designing instruction that recognizes and responds to the unique strengths and needs of each learner. This plan aims to strengthen teaching and learning by building cohesive systems of support for students, educators, and leaders.

Each section works to support **three key goals**:



## What is Numeracy?

As defined in this guidance, numeracy is the ability for all students to confidently understand, interpret, and apply mathematical concepts across all domains of mathematics in a variety of real-world and academic contexts.

### Numeracy IS...

- ✓ Encompassing of all grade levels and domains of mathematics
- ✓ Sustaining mastery of essential skills to make connections across domains
- ✓ Understanding mathematical concepts
- ✓ Interpreting mathematical information
- ✓ Applying math in real-world contexts
- ✓ Applying math in academic contexts
- ✓ Communicating mathematical thinking
- ✓ Choosing and using appropriate mathematical tools
- ✓ Problem solving in unfamiliar situations using mathematical reasoning
- ✓ For everyone

### Numeracy IS NOT...

- ✗ Solely computation/arithmetic
- ✗ Rote memorization
- ✗ Restricted to the math classroom
- ✗ Reserved for advanced math learners/ courses
- ✗ Reserved only for students who plan to attend college or enter STEM fields
- ✗ Disconnected from lived experiences
- ✗ A reflection of innate talent or fixed ability
- ✗ Solely for “math people”

Numeracy supports the development of a positive mathematical identity where students see themselves as capable, persistent, and confident problem solvers. Developing these abilities requires a strong mathematical foundation across the full K-12 continuum and supports the development of mathematically literate citizens. All students have the capacity to develop numeracy when they are provided with equitable access to high-quality instruction, culturally relevant learning experiences, and opportunities to engage meaningfully in mathematics.

## Why is This Guidance Important?

The [2025 Illinois Assessment of Readiness](#) results indicate that 38.5% of students in Grades 3–8 and 39.3% of high school students met grade-level proficiency standards in mathematics. This data is not a reflection of student potential. Instead, it highlights the need to closely examine and realign the systems and structures that guide mathematics instruction.

■ Below Proficient   
 ■ Approaching Proficient   
 ■ Proficient   
 ■ Above Proficient

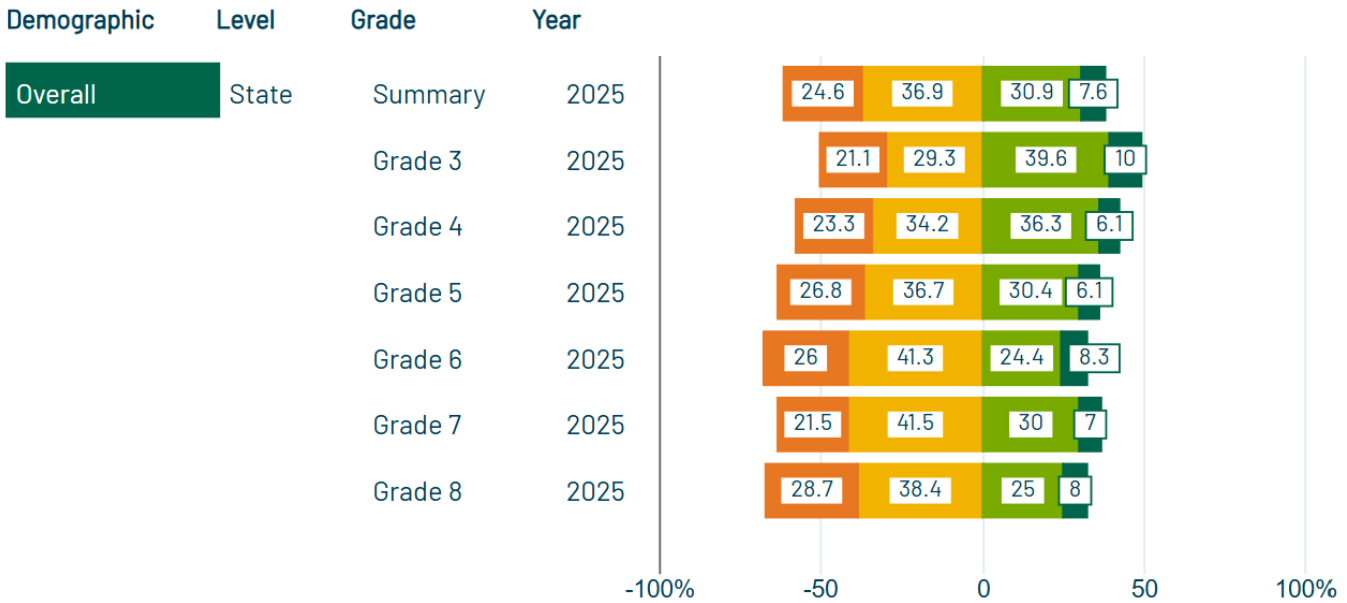


Figure 1: Percentage of students scoring at each of the performance levels for the Illinois Assessment of Readiness by year in 2025

■ Below Proficient   
 ■ Approaching Proficient   
 ■ Proficient   
 ■ Above Proficient

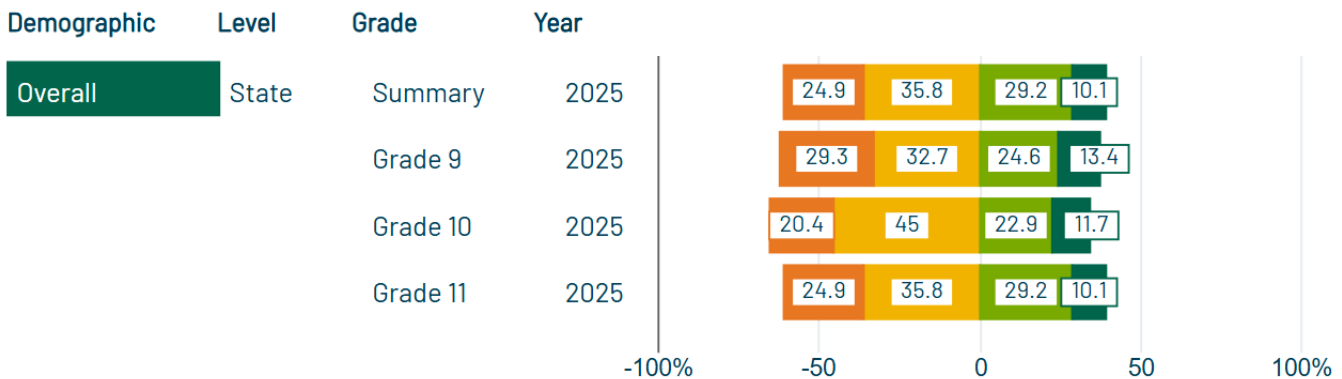


Figure 2: Percentage of students scoring at each of the performance levels in the ACT Suite by grade level in 2025

Recovery in mathematics continues to lag behind literacy, and persistent disparities endure across race, income, English proficiency, and disability status. Results from NAEP, Illinois SAT, and local benchmark assessments confirm these deficits and reflect systemic inequities in access to high-quality instruction, rigorous coursework, and culturally relevant learning opportunities. Math anxiety and negative perceptions of mathematics further compound the challenge for learners to succeed. Many people tend to either identify as “a math person” or as “not a math person,” an alarming classification given the significant role numeracy plays in long-term success and well-being.<sup>1</sup>



Test scores do not capture the full range of student strengths such as creativity, perseverance, character, or potential, but they do provide a starting point for action and must inform meaningful change. Numeracy extends far beyond academic success; it is a foundational life skill that shapes how individuals reason, solve problems, and make informed financial, health, and civic decisions. When students lack these skills, the consequences extend into adulthood, limiting access to opportunities and long-term well-being.<sup>2</sup> As renowned mathematician Andreas Schleicher states, “Good numeracy is the best protection against unemployment, low wages, and poor health,” while civil rights activist and educator Bob Moses reminds us that mathematics is a civil right that is essential for full participation in a democratic society, underscoring the critical role numeracy plays in shaping personal and societal well-being.

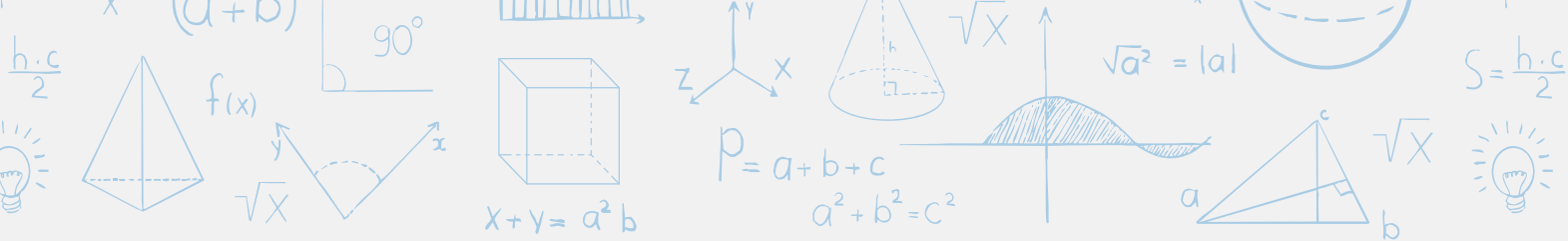
Prioritizing equitable, high-quality math instruction, culturally responsive teaching, and equitable access to resources is essential to ensure all students in Illinois can succeed in mathematics and in life. This guidance, rooted in evidence-based practices and data-driven decision making, aims to strengthen and align core mathematics instruction across the state.

### Building Conditions for Success

As the Why Math, Why Now? report highlights, mathematics serves as a gatekeeper to postsecondary success, career opportunities, and economic mobility, making systemic improvement essential.<sup>3</sup> Improving mathematics outcomes requires creating the right conditions for learning.



Research emphasizes that these systemic conditions play a vital role in advancing student success in mathematics.<sup>4</sup> The Illinois Comprehensive Numeracy Plan calls for statewide capacity-building, the expansion of evidence-based strategies, and assessment practices that value both conceptual understanding and procedural fluency. Additionally, this guidance provides an opportunity to chart a new course for numeracy in Illinois: one rooted in evidence, equity, and the belief that all students can develop the skills, habits of mind, and confidence needed to thrive mathematically. By focusing on proficiency, growth, and equity, Illinois can transform mathematics from a barrier into a pathway of opportunity for every learner.

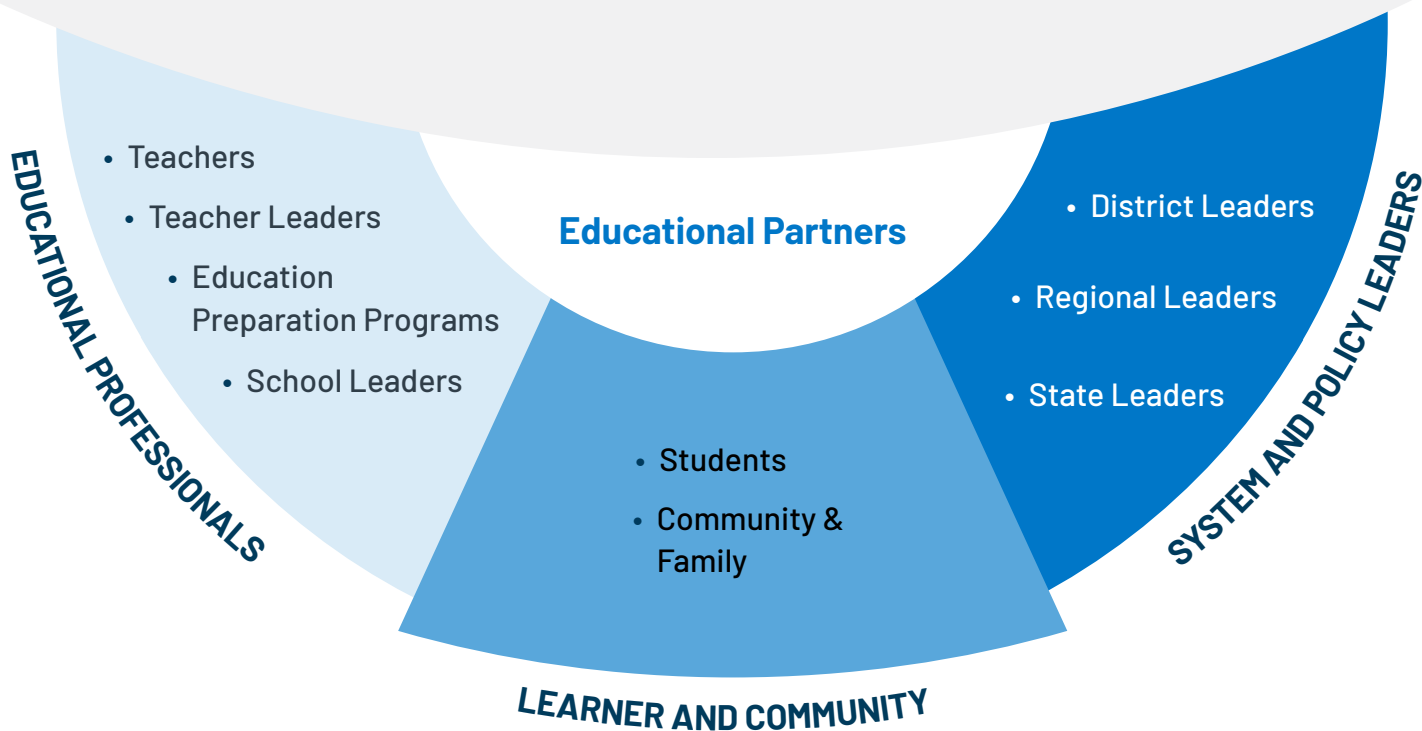


## Flexibility and Local Autonomy

ISBE provides guidance to districts rather than governance, imploring districts to either adopt the Numeracy Plan or create a local plan customized to their unique needs. The Numeracy Plan avoids prescribing specific materials or assessments while instead promoting data-informed choices by districts. This plan cannot address all district-specific contexts and challenges, and, therefore, ISBE encourages each district to carefully consider district and school-level data as it makes decisions related to numeracy instruction.

## Educational Partners

ISBE recognizes that each stakeholder group plays a pivotal role in numeracy education. ISBE believes in the shared responsibility of schools, families, and communities to help every student grow in mathematics and recognizes the value of working together with clear goals, a common language, and consistent practices. When all educational partners work in tandem, students benefit from a cohesive support network that promotes both academic and personal growth.



The following pages provide descriptions of educational partners and corresponding sections of note to support navigation of the plan.



## Educational Partner Descriptions

### Students

Students represent a wide range of mathematical experiences, backgrounds, and identities. They develop numeracy through opportunities to reason, explore, and make sense of mathematical ideas. As they grow, students build confidence by engaging in real-world problem solving and expressing their mathematical thinking in multiple ways.

### Teachers

Teachers include classroom educators, interventionists, and specialists who design daily learning experiences that build numeracy. They support students in developing conceptual understanding, procedural fluency, and mathematical reasoning. Teachers foster positive math identities and ensure equitable access to grade-level content.

### Teacher Leaders

Teacher leaders include math coaches, department chairs, bilingual specialists, and instructional leaders who support classroom teachers. They use data and content expertise to guide educators in implementing strong mathematics instruction. Teacher leaders help maintain coherence across grade levels and strengthen schoolwide numeracy practices.

#### Specific sections of note include:

- Vision and Purpose
- Section 1: Framework for the Evidence-Based Development of Numeracy Skills

Take an active role by exploring suggested resources, engaging in recommended instructional practices, and collaborating with educators to support individual learning needs.

#### Specific sections of note include:

- Vision and Purpose
- Section 1: Framework for the Evidence-Based Development of Numeracy Skills
- Section 2: Educator Professional Learning and Development
- Section 4: Tools and Resources

#### Specific sections of note include:

- Vision and Purpose
- Section 1: Framework for the Evidence-Based Development of Numeracy Skills
  - Cohesive Continuum of Learning
  - Six Components of Numeracy
  - Assessment
- Section 2: Educator Professional Learning and Development
- Section 4: Tools and Resources

## Educational Partner Descriptions

### School Leaders

School leaders, including principals and assistant principals, shape the conditions that support effective mathematics instruction. They guide curriculum implementation, professional learning, and collaborative planning. School leaders analyze data, promote equitable access to grade-level math, and ensure a positive culture for numeracy.

#### Specific sections of note include:

- Vision and Purpose
- Section 1: Framework for the Evidence-Based Development of Numeracy Skills
  - Cohesive Continuum of Learning
  - Six Components of Numeracy
  - Assessment
- Section 2: Educator Professional Learning and Development
- Section 3: Framework for Effective Leadership, Systems of Support, and Implementation Considerations
- Section 4: Tools and Resources

### District Leaders

District leaders such as school boards, superintendents, curriculum directors, and instructional administrators oversee systemwide numeracy efforts. They support curriculum adoption, assessment planning, and professional learning. District leaders analyze student data to guide decision-making and promote coherent, equitable mathematics instruction across schools.

#### Specific sections of note include:

- Vision and Purpose
- Section 1: Framework for the Evidence-Based Development of Numeracy Skills
  - Instructional Considerations
- Section 2: Educator Professional Learning and Development
- Section 3: Framework for Effective Leadership, Systems of Support, and Implementation Considerations
- Section 4: Tools and Resources

### Regional Leaders

Regional leaders, including ROEs and ISCs, provide multi-district support for improving numeracy. They offer professional learning, assist with curriculum review, and analyze regional data to identify needs. Regional leaders help ensure consistent, aligned implementation of evidence-based math practices across communities.

#### Specific sections of note include:

- Vision and Purpose
- Section 1: Framework for the Evidence-Based Development of Numeracy Skills
  - Evidence-Based Instruction
  - Cohesive Continuum of Learning
  - Six Components of Numeracy
  - Instructional Considerations
- Section 2: Educator Professional Learning and Development
- Section 3: Framework for Effective Leadership, Systems of Support, and Implementation Considerations
- Section 4: Tools and Resources



## Educational Partner Descriptions

### Educator Preparation Programs

Educator preparation faculty prepare future teachers to understand mathematical development and evidence-based instruction. They introduce candidates to learning progressions, content knowledge, and strategies that promote reasoning and conceptual understanding. These programs help develop confident, well-prepared mathematics educators.

#### Specific sections of note include:

- Vision and Purpose
- Section 1: Framework for the Evidence-Based Development of Numeracy Skills
- Section 2: Educator Professional Learning and Development

### State Leaders

State leaders, including ISBE and statewide agencies, establish the structures and guidance that support effective numeracy instruction. They monitor statewide data, develop aligned resources, and promote equity across districts. State leaders help create coherent systems that advance mathematics achievement for all students.

#### Specific sections of note include:

- Vision and Purpose
- Section 2: Educator Professional Learning and Development

### Community & Family

Families and community partners such as child care and tutoring centers are students' first and most enduring mathematical teachers. They connect numeracy to daily life, reinforce positive attitudes about learning math, and collaborate with schools to strengthen student success. When families and educators share a common language around numeracy, students benefit from a coherent system of support.

#### Specific sections of note include:

- Vision and Purpose
- Section 1: Framework for the Evidence-Based Development of Numeracy Skills

## Vision and Purpose End Notes

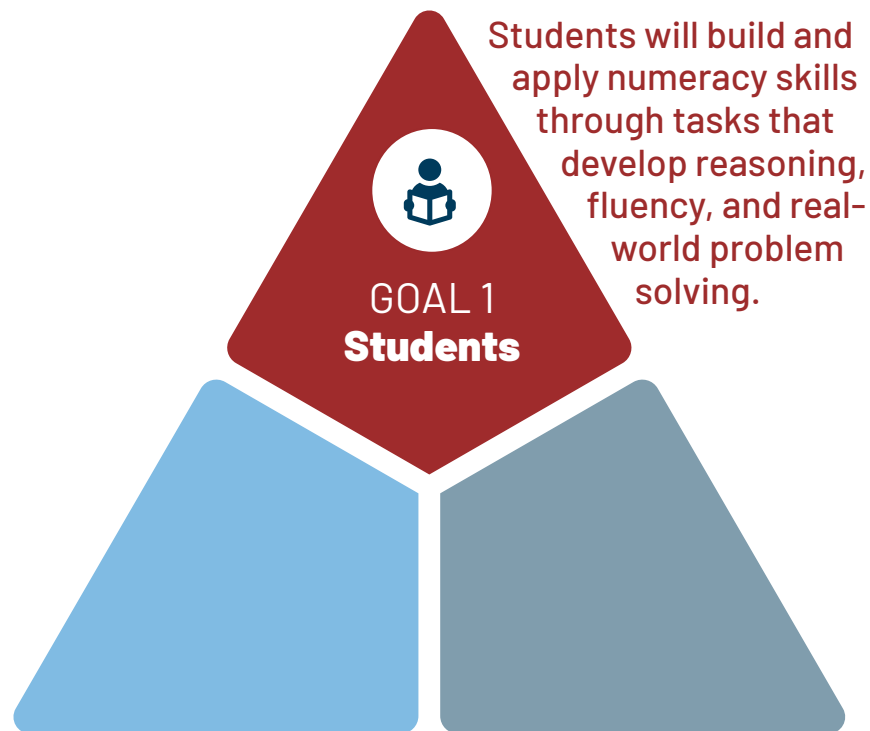
- <sup>1</sup> Parsons, Samantha, and John Bynner. [Does numeracy matter more?](#) 2005.
- <sup>2</sup> Organisation for Economic Co-operation and Development. [Skills Matter: Additional Results from the Survey of Adult Skills](#). Paris: OECD Publishing, 2019.
- <sup>3</sup> Bill & Melinda Gates Foundation. [Why Math, Why Now?](#) Seattle, WA: Gates Foundation, 2022.
- <sup>4</sup> Learning Policy Institute. [Positive Conditions for Mathematics Learning: An Overview of the Research](#). Palo Alto, CA: Learning Policy Institute, 2025.



# FRAMEWORK FOR THE EVIDENCE-BASED DEVELOPMENT OF NUMERACY SKILLS



This section is dedicated to **Goal 1:**



This section of the plan will cover the following main topics:

1. Evidence-Based Instruction

2. Cohesive Continuum of Learning

3. The Six Components of Numeracy

4. Instructional Considerations

5. Assessments

## Section Overview

This section begins with defining evidence-based mathematics instruction, then discusses the importance of coherent and aligned curriculum to avoid fragmented instruction and create a cohesive continuum for learning. Next, the plan offers insight into the development of mathematical skills and carefully lays out the Six Components of Numeracy. This section elaborates on instructional considerations that benefit all learners and provides considerations for learners with specialized needs. Lastly, this portion of the plan offers information about assessment and ends with workbook pages tailored to each audience type and reflection questions regarding Goal 1 of the plan.

## Evidence-Based Instruction

Evidence-based mathematics instruction draws on decades of research about how students learn. It is not a single program or method; it is a commitment to teaching in ways that a rigorous body of research consistently supports. As research advances and student demographics change, understandings of effective instruction evolve accordingly, making it essential to stay current with evidence-based best practices.

The Illinois Comprehensive Numeracy Plan follows the federal Every Student Succeeds Act (ESSA), endorsing instructional practices based on the Tiers of Evidence framework. This framework classifies practices into four evidence-based tiers that are determined by five factors:



\*The extent to which the characteristics of the study align with the characteristics of the organization that wishes to implement the educational program. Practices with strong (Tier 1) and moderate (Tier 2) evidence should be prioritized. This approach ensures that teaching methods are not just theoretically sound but also have proven effectiveness in enhancing student learning.



# UNDERSTANDING THE ESSA TIERS OF EVIDENCE



**Strong Evidence**



**Moderate Evidence**



**Promising Evidence**



**Demonstrates a Rationale**






	<b>TIER 1</b> Strong Evidence	<b>TIER 2</b> Moderate Evidence	<b>TIER 3</b> Promising Evidence	<b>TIER 4</b> Demonstrates a Rationale
 <b>Study Design</b>	Well-designed and implemented experimental study, meets WWC standards without reservations	Well-designed and implemented quasi-experimental study, meets WWC standards with reservations	Well-designed and implemented correlational study, statistically controls for selection bias <sup>a</sup>	Well-defined logic model based on rigorous research
 <b>Results of the Study</b>	Statistically significant positive effect on a relevant outcome	Statistically significant positive effect on a relevant outcome	Statistically significant positive effect on a relevant outcome	An effort to study the effects of the intervention is planned or currently under way
 <b>Findings From Related Studies</b>	No strong negative findings from experimental or quasi-experimental studies	No strong negative findings from experimental or quasi-experimental studies	No strong negative findings from experimental or quasi-experimental studies	N/A
 <b>Sample Size &amp; Setting</b>	At least 350 participants, conducted in more than one district or school	At least 350 participants, conducted in more than one district or school	N/A	N/A
 <b>Match</b>	Similar population <i>and</i> setting to your setting	Similar population <i>or</i> setting to your setting	N/A	N/A

Figure 3: Understanding the ESSA Tiers of Evidence

a. Findings from experimental and quasi-experimental studies that either (a) meet the first three criteria for Tiers 1 and 2 but not the sample size, setting, or match requirements, or (b) do not meet What Works Clearinghouse (WWC) standards but statistically control for selection bias between the treatment and comparison groups are also eligible to meet Tier 3 Promising Evidence. From *Institute of Education Sciences. [ESSA Tiers of Evidence](#). U.S. Department of Education, 2025.*

## Evidence-Based Instruction IS...

### ✓ **A Collection of Research to Inform Instruction**

Research about how children learn mathematics and what to do when a child encounters difficulty in learning mathematics. The research informs evidence-based instructional practices.

### ✓ **Ever-Evolving**

New evidence and research are continuously being released. As populations, communities, and approaches evolve, so should practice. New research can impact the weight of evidence. The continuum of rigor and quality for research can help identify the weight of claims stemming from research.

## Evidence-Based Instruction IS NOT...

### ✗ **A Program, Intervention, or a Product for Purchase**

The use of evidence-based instruction is an approach to teaching mathematics that is based on decades of research and evidence. It is not a specific program.

### ✗ **Complete and Final**

As with any research, it is never complete. Studies are constantly being conducted and new research continues to be released. Leaders, teachers, and families can work together to bring relevant evidence-based practices into the classroom.

## Mathematics Instructional Approaches

High-quality mathematics instruction is intentional, data-informed, and grounded in a coherent integration of evidence-based instructional approaches. Research in mathematics education indicates that effective instruction relies on the thoughtful blending of multiple methodologies, including explicit instruction, unassisted discovery, guided inquiry, guided strategic development, and strategic coaching.<sup>1</sup> The selection and integration of these approaches must be driven by ongoing analysis of student data to ensure alignment with learners' strengths, needs, and instructional goals. Positive learning outcomes are associated with these approaches when interwoven implementation is intentional and aligned to student needs and instructional goals.<sup>2</sup> High-quality instruction reflects a balanced use of teacher-directed and student-centered practices to support procedural fluency, conceptual understanding, and mathematical reasoning.<sup>3</sup>



## **Cohesive Continuum of Learning**

Numeracy skills develop over time as students build on prior knowledge and extend their understanding to new contexts.<sup>4</sup> A cohesive continuum of learning ensures that students experience mathematics as a connected progression of ideas rather than as disconnected topics. When learning is coherent, students are better able to make sense of mathematics, ultimately allowing them to make significant academic progress.<sup>5</sup>

Instructional coherence occurs when core instruction (including curriculum, assessments, and materials) and interventions are intentionally aligned to advance shared priorities, goals, and grade-level learning experiences for all students.<sup>6</sup> Students should encounter mathematical concepts multiple times across grade bands with increasing sophistication to foster students' confidence to apply mathematics flexibly and to transfer learning across domains and contexts.

To achieve this coherence, all mathematics instruction and assessments must be intentionally connected across grade levels and aligned to the Illinois Learning Standards for Mathematics. This includes both vertical alignment across grade levels and horizontal alignment within a grade level. This alignment ensures that students experience mathematics as a logical progression of ideas rather than isolated skills.<sup>7</sup> Vertical alignment also supports instructional coherence by allowing educators to understand how strategies and models are developed and used across grade levels, enabling them to make connections that help students build on prior learning and prepare for future concepts. Achieving such coherence requires educators to develop a deep understanding of the breadth and depth of the standards as well as a clear vision of learning within a grade, between grades, and within and between content strands. Additionally, standards should be understood as yearlong learning goals rather than isolated units of study. Instruction must be intentionally designed to ensure students are repeatedly exposed to and engage with all grade level standards.

The Illinois Learning Standards integrate conceptual understanding, procedural fluency, and application. This integration may occur within a single standard or develop across grade levels. For example, students' understanding of multiplication develops over time, beginning with early experiences such as sorting and evolves from grouping objects (concrete) to arrays and area models (pictorial), and eventually to symbolic algorithms (abstract) between second and sixth grade.<sup>8</sup> This concrete-pictorial-abstract (CPA) progression not only builds mathematical content knowledge but also shapes how students think and work mathematically.

These ways of thinking are articulated in the Standards for Mathematical Practice (SMPs), which describe the skills, dispositions, expertise, and understandings that all students should experience and develop. While the Illinois Learning Standards for Mathematics are grade-level specific in their identification of what students should know and be able to do, the eight Standards for Mathematical Practice are the same for each grade level in their description of the "habits of mind" that all students should develop as they progress through grade levels.<sup>9</sup> For example, the multiplication progression discussed above supports the development of flexible problem-solving skills and habits of mind, such as modeling (SMP 4), reasoning (SMP 3), and precision (SMP 6). These are the foundations for mathematical thinking and practice that are part of being a confident and proficient problem solver. These practices should permeate throughout the whole curriculum and as such are a unifying theme across the grades (i.e., the

practices can be thought of as the “heart and soul” of what it means to do mathematics). The Standards for Mathematical Practice are not discrete grade-level skills but enduring practices that deepen over time, meaning students should apply mathematical practices during the learning of each lesson throughout the K-12 continuum.<sup>10</sup> Teachers are charged with establishing an environment through which students engage with mathematical practices and providing meaningful experiences for the students to develop these habits of mind for themselves.

This example of multiplication demonstrates why it is critical for educators to be deeply familiar with grade level standards and learning progressions. Without this understanding, students may be introduced to concepts or procedures that are developmentally inappropriate, such as standard algorithms, before foundational understanding is established. Instruction must be intentionally sequenced so that students first explore ideas through hands-on experiences, then transition to visual or diagrammatic representations, and finally to symbolic or abstract reasoning. Students should have ongoing opportunities to revisit concrete and pictorial models to reinforce understanding or address misconceptions.

Finally, the selection of high-quality core instructional materials and supplemental instructional materials is essential to supporting alignment and coherence in mathematics education. Tools such as [EdReports](#), [ISBE’s Curriculum Evaluation Tool](#), and the [CCNetwork’s Guide to the Implementation of High-Quality Instructional Materials](#) provide frameworks to assist educators and school leaders in evaluating curriculum quality.



## Learning Standards

- [Counting & Cardinality](#) (K)
- [Number & Operations in Base Ten](#) (K-5)
- [Ratios & Proportional Relationships](#) (6-7)
- [Number & Quantity](#) (HS)
- [Number & Operations-Fractions](#) (3-5)
- [The Number System](#) (6-8)

- [Algebra](#) (HS)
- [Operations & Algebraic Thinking](#) (K-5)
- [Expressions & Equations](#) (6-8)
- [Modeling](#) (HS)
- [Functions](#) (8-12)
- [Geometry](#) (K-12)
- [Measurement & Data](#) (K-5)
- [Statistics & Probability](#) (6-12)

## Additional Resources

- [IELDS](#) (K)
- [KIDS](#) (K)
- [CTE](#) (HS)
- [Transitional Math](#) (HS)
- [Social Emotional Standards](#) (K-12)
- [WIDA](#) (K-12)

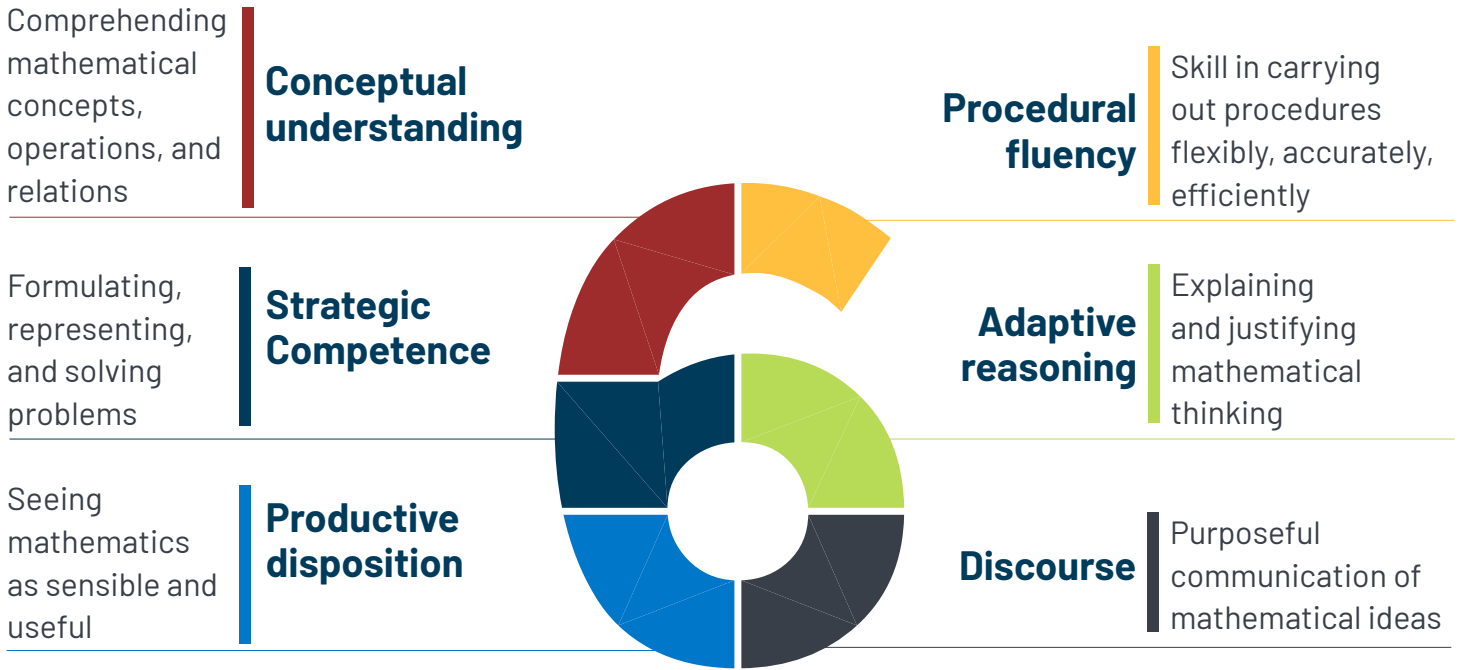


Grades										
K	1	2	3	4	5	6	7	8	High School	
Counting & Cardinality										
	Number & Operations in Base Ten					Ratios & Proportional Relationships			Number & Quantity	
			Number & Operations Fractions			The Number System			Algebra	
<b>Domains &amp; Standards</b>	Operations & Algebraic Thinking					Expressions & Equations		Modeling		
								Functions		
	Geometry									
	Measurement & Data					Statistics & Probability				
IELDS									CTE	
KIDS									Transitional Math	
<b>Additional Resources</b>	Social Emotional Standards									
	WIDA									

Figure 4: Learning Standards

# The Six Components of Numeracy

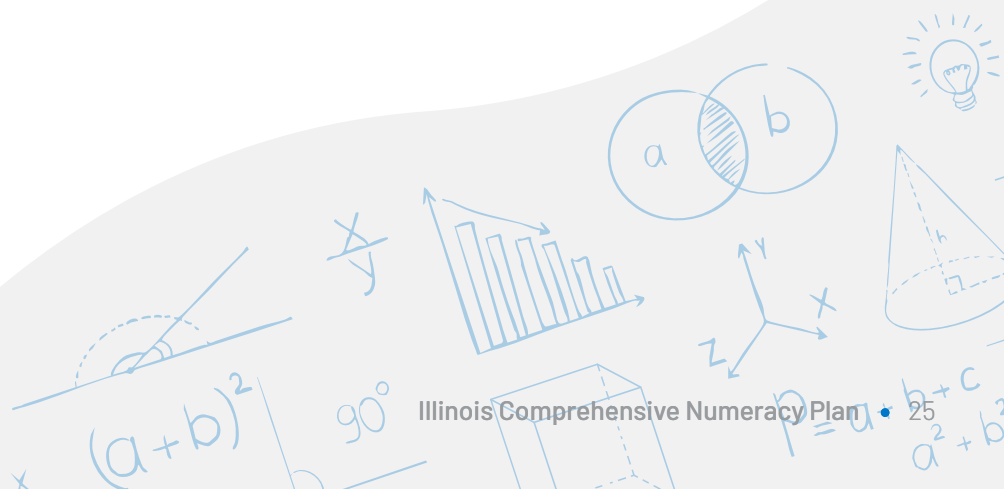
Illinois has identified six components of numeracy detailed on the following pages.



The components, with the exception of discourse, were established by the National Research Council and offer a comprehensive overview of instructional practices. These practices align with the crucial components of numeracy. The instructional framework is not merely a theoretical construct; it is a practical tool for educators that is designed to enhance numeracy teaching and learning throughout Illinois.

The Six Components of Numeracy are not independent of one another; thus, they are not isolated skills. Each component works together to cultivate numeracy in students, which is a vital skill for success in both students' personal and professional lives. Effective mathematics instruction incorporates each component to build numeracy in students.

The following pages delve deeper into the Six Components of Numeracy. Each component is accompanied by "Instructional Practices and Strategies". The practices in these tables are specific to core, Tier 1 instruction. Not all practices are appropriate across all grade levels, as effectiveness varies by context. This list is not exhaustive and does not include evidence-based interventions designed for more targeted or intensive supports beyond core instruction. Professional judgement remains essential in determining how best to meet the diverse needs of students.



## Conceptual understanding

Conceptual understanding is the ability for students to view mathematical concepts in a connected, meaningful way, recognizing their significance and organizing knowledge so that new concepts are linked to their existing understanding.<sup>11</sup> Benefits of conceptual understanding include increased retention rates of mathematical concepts, fewer computational errors, and students' ability to flexibly apply their knowledge, which ultimately develops their critical-thinking skills.<sup>12</sup> A strong conceptual understanding helps students develop a deep understanding of math concepts and their interconnectedness and allows them to approach math problems with a variety of strategies.



### Why It Matters

Conceptual understanding is essential in mathematics because it enables students to make sense of mathematical ideas, relationships, and structures rather than relying on memorized procedures. When students understand the “why” behind the mathematics, they are better equipped to interpret and act on the quantitative information they encounter in everyday life such as comparing prices, managing a budget, evaluating interest rates, or making sense of data presented in the media. This depth of understanding allows students to translate real-world situations into mathematical representations and to interpret those results in ways that inform decisions. By engaging in authentic contexts, students learn to recognize patterns, evaluate claims, and determine whether results are reasonable, strengthening their ability to think critically in a data-rich world. These experiences position mathematics as a practical tool for problem solving and informed decision-making rather than a set of isolated skills. Ultimately, conceptual understanding prepares students to navigate personal, civic, and professional situations with confidence and sound reasoning.

- ✓ Supports informed decision-making in everyday contexts (e.g., budgeting, pricing, rates, and risk)
- ✓ Strengthens the ability to interpret and evaluate data encountered in media and daily life
- ✓ Builds durable understanding that transfers across contexts
- ✓ Reduces reliance on rote memorization and isolated procedures
- ✓ Connects multiple representations (e.g., graphs, equations, tables, models)
- ✓ Serves as the foundation for procedural fluency and continued learning



## Conceptual understanding

### Instructional Practices and Strategies<sup>13</sup>

#### Aligned with Evidence

- ✓ Use of concrete-pictorial-abstract (CPA) progression
- ✓ Emphasis on multiple representations (models, visuals, symbols)
- ✓ Tasks that require explanation of “why it works”
- ✓ Use of number talks and math routines
- ✓ Connecting prior knowledge to new concepts
- ✓ Building understanding through schema recognition
- ✓ Encouraging student-generated models and representations
- ✓ Formative questioning focused on relationships, not answers
- ✓ Use of rich tasks with multiple entry points

#### Not Aligned with Evidence

- ✗ Overemphasis on memorization without conceptual grounding
- ✗ Isolated skill practice
- ✗ Single-representation instruction only
- ✗ Premature abstraction without concrete support
- ✗ Emphasis on speed over understanding
- ✗ Tasks with only one correct representation
- ✗ Teacher-only modeling without student construction of ideas
- ✗ Lack of explicit connections between concepts

### Specialized Learning Considerations

#### Multilingual Learners

- ✓ Visual models paired with vocabulary supports
- ✓ Hands-on manipulatives for meaning making
- ✓ Language-rich discussion anchored in visuals

#### Students with IEPs/504 Plans

- ✓ Chunking concepts and tasks
- ✓ Step-by-step visual scaffolds (graphic organizers, number lines, etc.)
- ✓ Extended time with manipulatives before abstraction

#### Gifted & Advanced Learners

- ✓ Open-ended “what if” explorations
- ✓ Multiple representations of the same concept
- ✓ Generalization and pattern exploration tasks

#### Students with Dyscalculia

- ✓ Heavy reliance on concrete models and manipulatives
- ✓ Explicit teaching of number relationships
- ✓ Reduced symbolic overload early in learning

## Conceptual understanding

SMPs	Teacher Actions	Student Actions
MP.2 MP.4 MP.7	<ul style="list-style-type: none"> <li>✓ Make mathematics accessible to students through concrete, pictorial, and abstract representations</li> <li>✓ Explicitly teach and name problem schemas</li> <li>✓ Proactively facilitate multiple “access points/modes” for students</li> <li>✓ Use multiple representations when appropriate while using and eliciting student thinking</li> <li>✓ Ask students to provide visual proof of their solution</li> </ul>	<ul style="list-style-type: none"> <li>✓ Demonstrate understanding of concepts in multiple ways</li> <li>✓ Make multiple connections among different models and representations</li> <li>✓ Justify strategy choices based on problem structure rather than key words</li> <li>✓ Understand that multiple iterations may be needed to reach an appropriate solution and persevere in the revisions of thinking</li> <li>✓ Use multiple modes of communication to convey solutions and explain what the solutions mean in context</li> </ul>





### Assessment Considerations

Assessment of conceptual understanding should focus on revealing how students reason, make connections, and apply ideas—not just whether they can produce the correct answers. Effective assessments include open-ended tasks and rich problems that require explanation, representation, and justification of thinking. Students should be asked to use multiple representations (models, diagrams, symbols, words) to demonstrate understanding of underlying concepts and relationships. Formative assessment plays a critical role, providing ongoing evidence of learning that informs instructional pivots and next steps. Teachers should attend to students’ misconceptions and partial understandings, using assessment data to support targeted feedback and instructional responses. Opportunities for student self-assessment and reflection help learners articulate what they understand and where they are still developing. Assessment should be coherent with instruction, aligned to learning goals, and embedded regularly, not reserved only for summative measures. High-quality assessment practices emphasize conceptual growth, reasoning, and sense-making as central indicators of mathematical proficiency.

### Progressions

#### Young Learners: Concrete Foundations

- ✓ Develop number sense through manipulatives and visual models
- ✓ Understand quantity, comparison, and basic operations conceptually

**Example:** “Show different ways to make 10 using objects or drawings.” (K.OA)

#### Elementary: Expanding Relationships

- ✓ Understand operations as relationships (inverse, equivalence)
- ✓ Use area models, arrays, and number line

**Example:** “Explain how multiplication is related to repeated addition using an array.” (3.OA)

#### Middle School: Abstract Connections

- ✓ Generalize relationships across numbers, ratios, and expressions
- ✓ Connect algebraic representations to visual models

**Example:** “Explain why multiplying by a fraction less than 1 reduces a quantity.” (6.NS)

#### High School: Structural Understanding

- ✓ Understand structure of expressions, functions, and transformations
- ✓ Reason about equivalence and generalization

**Example:** “Interpret the meaning of parameters in a linear function in context.” (F-IF)

## Procedural fluency

Procedural fluency extends beyond the ability to perform standard algorithms and involves understanding mathematical processes and procedures, knowing when and how to apply them effectively, and being able to execute them with flexibility, accuracy, and efficiency.<sup>14</sup> Procedural fluency also includes the ability to accurately estimate and check for reasonableness, which closely aligns with the development of conceptual understanding. A key component of procedural fluency is automaticity, the ability to recall and execute basic facts and procedures with little cognitive effort, which supports efficient problem solving and frees students' working memory to focus on reasoning and underlying concepts.<sup>15</sup> A student's ability to perform mental mathematics calculations with little to no errors lays the foundation for higher math as well as allows students to focus on the "why" behind the procedure. Procedural fluency builds the foundation for higher order thinking, advanced mathematics skills, and real-world applications such as budgeting, calculating a tip at a restaurant, time management, etc. Ultimately, students who are fluent with procedures can easily apply the appropriate strategies to produce the correct answers or reasonable estimates and understand they must remain flexible in their approach to problem solving, as mathematics strategies are not a one-size-fits-all solution.



### Why It Matters

Procedural fluency is essential in mathematics because it enables students to carry out calculations and processes accurately, efficiently, and flexibly. In everyday life, individuals rely on these skills to manage tasks such as calculating totals, determining discounts and tips, adjusting recipes, measuring materials, or interpreting quantities like medication dosages where precision matters. When students develop fluency through meaningful application rather than isolated repetition, they not only perform procedures correctly but also understand when and why to use them. This efficiency reduces cognitive load, allowing individuals to focus on problem solving, reasoning, and making informed decisions rather than getting stuck in computation. Recognizing patterns and structures such as using place value, properties of operations, or mental math strategies further streamlines thinking and increases flexibility across contexts. As a result, procedural fluency becomes a practical, transferable skill that supports accuracy, confidence, and independence in daily life. Ultimately, it ensures mathematics functions as a reliable and efficient tool for real-world decision-making.

- ✓ Enables efficient and accurate problem solving in everyday contexts
- ✓ Frees cognitive capacity for reasoning, modeling, and decision-making
- ✓ Reduces errors and supports precision in real-world tasks
- ✓ Strengthens flexibility through pattern recognition and strategic thinking
- ✓ Supports confident, independent use of mathematics in daily situations



## Procedural fluency

### Instructional Practices and Strategies<sup>16</sup>

#### Aligned with Evidence

- ✓ Number talks and mental math routines
- ✓ Emphasis on strategy comparison before formal algorithms
- ✓ Explicit connection between concept and procedure
- ✓ Incorporate spaced retrieval and fluency practice
- ✓ Use of error analysis tasks
- ✓ Encouraging multiple solution pathways
- ✓ Spiral review embedded in meaningful contexts
- ✓ Visual models to support procedures
- ✓ Timely, specific feedback on reasoning
- ✓ Tasks that require selecting appropriate strategies

#### Not Aligned with Evidence

- ✗ High stakes timed tests as primary fluency measure
- ✗ Memorization without conceptual grounding
- ✗ Isolated skill drills without context
- ✗ Teaching procedures before conceptual understanding
- ✗ Single-method instruction only
- ✗ Emphasis on speed over accuracy or reasoning
- ✗ Lack of student explanation of procedures
- ✗ No opportunity for strategy comparison
- ✗ Procedures taught as “rules” or “steps to follow” without meaning

### Specialized Learning Considerations

#### Multilingual Learners

- ✓ Visual step charts and bilingual supports
- ✓ Modeling procedures with language scaffolds

#### Students with IEPs/504 Plans

- ✓ Reduced problem sets with high-quality feedback
- ✓ Step-by-step guided practice

#### Gifted & Advanced Learners

- ✓ Efficiency comparisons and optimization tasks
- ✓ Exploration of non-standard methods

#### Students with Dyscalculia

- ✓ Manipulatives to anchor procedures
- ✓ Structured repetition with conceptual reinforcement

## Procedural fluency

SMPs	Teacher Actions	Student Actions
MP.6 MP.7 MP.8	<ul style="list-style-type: none"> <li>✓ Provide opportunities for distributed practice of procedures</li> <li>✓ Provide multiple opportunities using a variety of formats to assess students' understanding</li> <li>✓ Provide opportunities for students to connect student-generated strategies with efficient procedures</li> <li>✓ Have students play games that involve strategy and the use of procedural fluency</li> <li>✓ Recognize the need for various modes of explanation, including written, verbal, and diagrams</li> <li>✓ Uses precise mathematical language when describing strategies</li> <li>✓ Provide feedback based on student thinking</li> <li>✓ Provide opportunities for students to find and analyze errors</li> </ul>	<ul style="list-style-type: none"> <li>✓ Accurately use a procedure on an unfamiliar problem or new number set</li> <li>✓ Demonstrate flexibility in using strategies and recognize and justify when one strategy is more efficient than others</li> <li>✓ Identify, use, and communicate multiple pathways to a solution</li> <li>✓ Find and analyze errors to show an understanding of the procedures and why they work</li> </ul>





### Assessment Considerations

Assessment of mathematical procedural understanding should focus on how accurately, efficiently, and flexibly students carry out mathematical procedures. Effective assessments evaluate not only correct answers but also students' ability to select and apply appropriate procedures in varied contexts. Tasks should examine procedural fluency, including accuracy, efficiency, and consistency over time, while avoiding an exclusive focus on speed. Assessments benefit from including non-routine problems that require students to adapt procedures rather than follow memorized steps. Teachers should use formative checks (such as worked examples, exit tasks, and error analysis) to identify gaps in fluency or misunderstandings that hinder procedural use. Attention to common errors and misconceptions helps distinguish procedural mistakes from deeper conceptual gaps. Feedback should be specific and corrective, supporting refinement and automaticity of procedures.

### Progressions

#### Young Learners: Concrete Foundations

- ✓ Develop efficient counting strategies and number relationships

**Example:** "Solve  $7+5$  using a make-ten strategy." (1.OA)

#### Elementary: Expanding Relationships

- ✓ Use standard algorithms and invented strategies
- ✓ Understand why algorithms work

**Example:** "Solve  $256 \times 4$  using two different strategies." (4.NBT)

#### Middle School: Abstract Connections

- ✓ Work fluently with rational numbers and expressions

**Example:** "Solve multi-step equations and justify each step." (7.EE)

#### High School: Structural Understanding

- ✓ Manipulate algebraic, exponential, and function expressions

**Example:** "Rewrite expressions to reveal key features of a function." (A-SSE)

## Strategic competence

### Strategic Competence

Strategic competence is the capacity to formulate, represent, and solve mathematical problems effectively.<sup>17</sup> It includes problem solving as well as problem formulation and recognizes that real-world scenarios often require students to define the problem before applying a mathematical strategy to solve the problem. The development of strategic competence requires practicing problem formulation, cultivating a repertoire of math strategies, and understanding which strategies are appropriate for a given scenario. Strategic competence allows students to become proficient problem solvers and has strong ties to procedural fluency and conceptual understanding.



### Why It Matters

Strategic competence is essential in mathematics because it enables students to formulate, represent, and solve problems with purpose and flexibility. In everyday life, individuals draw on this skill when making decisions such as comparing long-term costs, planning travel based on time and budget, or prioritizing expenses to manage finances effectively. Rather than relying on memorized steps, students with strong strategic competence can analyze situations, choose appropriate approaches, and adjust their thinking when initial strategies are unsuccessful. They also learn to use tools intentionally, such as estimation for quick decisions, digital tools to model scenarios, or diagrams and tables to organize and compare information. This adaptability allows individuals to navigate complex, real-world situations where there is no single clear path to a solution. By integrating conceptual understanding and procedural fluency, strategic competence supports thoughtful, efficient problem solving across contexts. Ultimately, it empowers students to use mathematics as a tool for planning, evaluating options, and making informed decisions in daily life.

- ✓ Develops independent and adaptable problem solvers
- ✓ Strengthens transfer of mathematical thinking across contexts
- ✓ Builds decision-making skills in real-world situations
- ✓ Supports effective use of tools and representations
- ✓ Integrates conceptual understanding and procedural fluency for purposeful problem solving



### Instructional Practices and Strategies <sup>18</sup>

#### Aligned with Evidence

- ✓ Use of non-routine, low-floor/high-ceiling tasks
- ✓ Teaching problem-solving strategies
- ✓ Encouraging multiple representations (table, graph, equation)
- ✓ Thinking routines (read-represent-solve-check)
- ✓ Collaborative problem-solving structures
- ✓ Anchoring tasks in real-world contexts
- ✓ Emphasis on explanation of strategy choice
- ✓ Use of estimation before solving
- ✓ Regular exposure to multi-step problems
- ✓ Reflection on problem-solving process

#### Not Aligned with Evidence

- ✗ Routine-only problems with predictable structures
- ✗ Teaching “key words” as primary strategy
- ✗ Over-scaffolding
- ✗ Tasks without real-world or conceptual context
- ✗ Single-representation problem solving
- ✗ Focus on answer over process
- ✗ Lack of discussion of strategy selection
- ✗ Minimal student reasoning required

### Specialized Learning Considerations

#### Multilingual Learners

- ✓ Visual problem breakdowns and vocabulary supports
- ✓ Structured partner talk for comprehension

#### Students with IEPs/504 Plans

- ✓ Chunked problems with guided steps
- ✓ Reduced linguistic complexity

#### Gifted & Advanced Learners

- ✓ Open-ended or multiple-solution problems
- ✓ Extension through constraints or generalization

#### Students with Dyscalculia

- ✓ Visual modeling and manipulatives
- ✓ Reduced memory load through structured supports

## Strategic competence

SMPs	Teacher Actions	Student Actions
MP.1 MP.5	<ul style="list-style-type: none"> <li>✓ Develop content knowledge to utilize multiple strategies and progressions</li> <li>✓ Develop the pedagogical knowledge to allow for multiple ways of solving problems</li> <li>✓ Provide class discussions on the efficiency of different strategies through questioning</li> <li>✓ Anticipate barriers or misconceptions that may arise during instruction and create intentional questions to engage and develop strategies</li> <li>✓ Provide a thinking task or problem that can be solved in multiple ways with a specific instructional goal in mind</li> <li>✓ Provide feedback on student processes and strategies</li> </ul>	<ul style="list-style-type: none"> <li>✓ Ask peers about strategy selected to compare with own</li> <li>✓ Reflect on which strategies are best used and when to use them when looking at work</li> <li>✓ Recognize that if arriving at a solution that does not make sense, there is another strategy to try</li> <li>✓ Formulate mathematical problems, represent, and solve them</li> <li>✓ Explain other students' solution strategies and connect to own</li> </ul>





### Assessment Considerations

Assessment of strategic competence should focus not only on correct solutions but also on how students analyze problems, select appropriate strategies, and adapt their approach when faced with challenges. Tasks aligned to grade-level mathematics standards should require students to determine what mathematics is needed rather than telling students which procedure to apply, ensuring alignment with the intent of the standards. Performance tasks, open-ended problems, and real-world scenarios allow students to demonstrate strategic competence by showing multiple solution paths, creating representations, and justifying their decisions. Rubrics should value strategy selection, evidence of reasoning, and flexibility alongside accuracy. Together, these assessment considerations provide a more complete picture of student understanding and ensure that strategic competence is developed and measured in ways that reflect both rigorous math standards and authentic mathematical thinking.

### Progressions

#### Young Learners: Concrete Foundations

- ✓ Use objects and drawings to solve problems

**Example:** “Solve: You have 6 apples and get 3 more. How many apples do you have now?” (1.OA)

#### Elementary: Expanding Relationships

- ✓ Choose and apply strategies for word problems

**Example:** “Solve a multi-step word problem involving fractions.” (5.NF)

#### Middle School: Abstract Connections

- ✓ Translate real-world problems into equations and models

**Example:** “Write an equation that represents a proportional relationship.” (7.RP)

#### High School: Structural Understanding

- ✓ Build and analyze models for real-world situations

**Example:** “Model exponential growth and interpret parameters.” (F-LE)

## Adaptive reasoning

▶ Adaptive reasoning is the ability to think logically about the connections among concepts and situations, evaluate alternative approaches to problem solving, and justify solutions.<sup>19</sup> In mathematics, it serves as the guiding principle that integrates facts, procedures, and ideas into a coherent whole. Students who successfully develop adaptive reasoning apply prior knowledge to justify their answers through reasoning rather than rely on confirmation from teachers or peers.



### Why It Matters

Adaptive reasoning is essential in mathematics because it enables students to think logically, reflect on their reasoning, and justify their conclusions. In everyday life, individuals rely on this skill to determine whether a solution is reasonable, reconsider decisions when conditions change, and explain or defend their thinking. Adjusting a budget after an unexpected expense, interpreting data in the news, or modifying plans when outcomes differ from expectations all require adaptive reasoning skills. Students with strong adaptive reasoning do more than arrive at answers; they evaluate the validity of their solutions, recognize errors, and revise their approach when needed. This ability to reflect and adjust strengthens both understanding and problem solving, allowing individuals to respond thoughtfully in situations where information is incomplete or evolving. By connecting concepts and reasoning, adaptive reasoning supports deeper learning and long-term retention. It also promotes equitable access by positioning all students as capable thinkers whose ideas can be examined, refined, and justified. Ultimately, adaptive reasoning ensures that mathematics is not only used but trusted as a tool for making informed decisions.

- ✓ Develops logical and critical thinking skills
- ✓ Supports justification, explanation, and mathematical argumentation
- ✓ Strengthens connections between concepts and procedures
- ✓ Enables evaluation of solutions and identification of errors
- ✓ Builds independence and confidence in mathematical thinking



### Instructional Practices and Strategies<sup>20</sup>

#### Aligned with Evidence

- ✓ Routine use of “justify your answer” prompts
- ✓ Structured argumentation tasks (claim-evidence-reasoning)
- ✓ Encouraging critique of multiple solution methods
- ✓ Use of error analysis as a reasoning tool
- ✓ Emphasis on explaining “why it works,” not just “how”
- ✓ Incorporation of visual and symbolic justification
- ✓ Peer discourse focused on reasoning validation
- ✓ Use of true/false and always/sometimes/never tasks
- ✓ Teacher facilitation that probes reasoning depth

#### Not Aligned with Evidence

- ✗ Accepting answers without justification
- ✗ Focusing only on procedural correctness
- ✗ Limited opportunities for student explanation
- ✗ Teacher provides reasoning instead of students constructing it
- ✗ Overuse of recall-based questioning
- ✗ Tasks that do not allow for multiple reasoning paths
- ✗ Emphasis on speed over explanation
- ✗ Lack of critique or comparison of methods
- ✗ Reasoning treated as optional rather than expected
- ✗ Minimal use of mathematical language in explanations

### Specialized Learning Considerations

#### Multilingual Learners

- ✓ Sentence frames for justification (“I know this because...”)
- ✓ Visual models paired with oral explanation
- ✓ Partner-supported reasoning discussions

#### Students with IEPs/504 Plans

- ✓ Scaffolded argument structures (sentence starters, templates)
- ✓ Reduced linguistic complexity in reasoning tasks
- ✓ Oral rather than written justification options

#### Gifted & Advanced Learners

- ✓ Formal proof and generalization tasks
- ✓ “Always/sometimes/never” justification extensions
- ✓ Student-led critique and debate opportunities

#### Students with Dyscalculia

- ✓ Emphasis on verbal and visual reasoning over symbolic load
- ✓ Use of manipulatives to support logic building
- ✓ Step-by-step reasoning scaffolds

## Adaptive reasoning

SMPs	Teacher Actions	Student Actions
MP.2 MP.3	<ul style="list-style-type: none"> <li>✓ Make connections to prior knowledge to further student thinking</li> <li>✓ Utilize protocols for students to justify their mathematical claims and respond to those of others</li> <li>✓ Anticipate student strategies and plan responses</li> <li>✓ Acknowledge, listen, and respond to student thinking and revisit when needed</li> <li>✓ Create a classroom culture where sharing initial thinking, think-alouds, and rough drafts are valued</li> </ul>	<ul style="list-style-type: none"> <li>✓ Find and correct errors, recognizing that mistakes should be used to further development of the concept</li> <li>✓ Identify multiple strategies to approach a problem and justify the approach taken</li> <li>✓ Justify the change in the initial approach to a problem</li> <li>✓ Provide informal and formal explanations and justifications</li> <li>✓ Apply what has been learned from one idea, task, etc., to a new scenario</li> <li>✓ Ask and answer questions to clarify the task</li> </ul>





### Assessment Considerations

To capture adaptive reasoning, assessments should intentionally include opportunities to reason abstractly and quantitatively (SMP 2) and construct viable arguments and critique reasoning (SMP 3). Examples include open-ended problems that ask students to explain their thinking, error-analysis tasks where students identify and correct a flawed solution, and “what-if” questions that require students to adjust a solution when conditions change. Performance tasks, such as interpreting real-world data, revising a model based on new constraints, or comparing two solution strategies and justifying which is more reasonable, provide strong evidence of adaptive reasoning. Rubrics should value clarity of explanation, logical coherence, accuracy, and the ability to revise thinking. Together, these assessment approaches ensure that adaptive reasoning is measured authentically, emphasizing flexibility, justification, and reflective thinking as key components of mathematical proficiency.

#### Progressions

##### Young Learners: Concrete Foundations

- ✓ Explain thinking using objects, drawings, and language
- ✓ Identify patterns and simple relationships

**Example:** “Explain why 12 is even using objects or drawings.” (2.OA)

##### Elementary: Expanding Relationships

- ✓ Use models and properties to explain reasoning
- ✓ Compare and critique simple solution strategies

**Example:** “Explain why two fractions are equivalent using a visual model.” (4.NF)

##### Middle School: Abstract Connections

- ✓ Construct and critique mathematical arguments
- ✓ Use definitions, properties, and prior knowledge

**Example:** “Justify whether a given equation has one, none, or infinitely many solutions.” (8.EE)

##### High School: Structural Understanding

- ✓ Develop formal proofs and logical arguments
- ✓ Justify assumptions and evaluate validity of claims

**Example:** “Prove properties of triangles using geometric reasoning.” (G-CO)

## Productive disposition

### Productive Disposition

Productive disposition refers to a positive belief that mathematics makes sense, is valuable, and can be mastered through perseverance. It also includes confidence in students' capacity to learn and use mathematics correctly.<sup>21</sup> Productive disposition develops alongside all other components of numeracy, as all components work together to create competent, confident mathematics learners. Teachers play a crucial role in developing productive dispositions among students. It is paramount that teachers model productive dispositions and create learning conditions and opportunities that promote students who view themselves as capable problem solvers who understand that mathematics connects to the world around them in infinite ways.



### Why It Matters

Productive disposition is essential in mathematics because it reflects a student's belief that mathematics is sensible, useful, and worth the effort. In everyday life, individuals regularly encounter situations that require persistence and flexibility, such as sticking with a budget when costs change, revising plans when initial calculations do not work out, or working through multi-step decisions like planning travel or managing time and resources. When students develop a productive disposition, they are more willing to engage with complex problems, persist through challenges, and view effort as a pathway to understanding rather than a barrier. This mindset supports the development of positive mathematical identity and reduces avoidance behaviors by helping learners see themselves as capable problem solvers. Through meaningful, real-world tasks, students experience mathematics as relevant and useful, which increases motivation to persist even when solutions are not immediately clear. Over time, these experiences build resilience, confidence, and independence in mathematical thinking. Ultimately, productive disposition empowers students to approach both academic and real-world challenges with perseverance and a belief in their ability to succeed.

- ✓ Encourages persistence in real-world problem solving and decision-making
- ✓ Builds a positive mathematical identity and sense of agency
- ✓ Reduces avoidance behaviors and supports confidence with challenge
- ✓ Strengthens engagement in complex, multi-step tasks
- ✓ Reinforces the belief that effort leads to understanding and success



### Instructional Practices and Strategies<sup>22</sup>

#### Aligned with Evidence

- ✓ Normalize making mistakes as part of learning
- ✓ Praise effort, strategy, and reasoning (not speed or correctness alone)
- ✓ Use open-ended and low-floor/high-ceiling tasks
- ✓ Provide time for revision and reflection
- ✓ Use collaborative problem-solving structures
- ✓ Explicitly teach persistence strategies (“try a different representation”)
- ✓ Provide feedback focused on process, not just answers
- ✓ Build classroom norms around mistake-making as learning
- ✓ Use math discussions to validate multiple approaches

#### Not Aligned with Evidence

- ✗ Overemphasis on correct answers over process
- ✗ Immediate teacher rescue during struggle moments
- ✗ Rewarding speed over persistence
- ✗ Avoidance of challenging tasks
- ✗ Excessive repetition without cognitive demand
- ✗ Discouraging mistakes or correcting too quickly
- ✗ Limited reflection on learning process
- ✗ Lack of opportunities for revision or rethinking
- ✗ Task design that limits challenge or exploration

### Specialized Learning Considerations

#### Multilingual Learners

- ✓ Encourage oral participation before written output
- ✓ Provide culturally responsive math contexts
- ✓ Use visuals to reduce language barriers

#### Students with IEPs/504 Plans

- ✓ Break tasks into manageable steps with checkpoints
- ✓ Provide structured encouragement for persistence
- ✓ Allow alternative demonstration of understanding

#### Gifted & Advanced Learners

- ✓ Provide extended challenge tasks requiring persistence
- ✓ Encourage exploration beyond initial solution
- ✓ Allow student-designed investigations

#### Students with Dyscalculia

- ✓ Reduce cognitive overload through scaffolding
- ✓ Use manipulatives to support sustained engagement
- ✓ Provide frequent success points to maintain confidence

## Productive disposition

SMPs	Teacher Actions	Student Actions
MP.1	<ul style="list-style-type: none"> <li>✓ Believe every student is capable of learning mathematics</li> <li>✓ Develop perseverance in students through complex tasks and encouragement</li> <li>✓ Provide opportunities for students to grow and show mastery by connecting math learned in the classroom to their community and the world</li> <li>✓ Utilize protocols for students to share their thinking and make sense of others' thinking</li> <li>✓ Model utilizing mistakes and multiple strategies through think-alouds to demonstrate the learning process</li> <li>✓ Allow students opportunities to identify and correct errors</li> </ul>	<ul style="list-style-type: none"> <li>✓ Believe that math can be used to solve problems in the community and the world</li> <li>✓ Persevere in learning new concepts, recognizing that understanding concepts takes time</li> <li>✓ Accept the challenge of the task and recognize that there is always a place to start when trying to solve a problem</li> <li>✓ Recognize that there are a variety of strategies to choose from when approaching a math problem</li> <li>✓ Share own thinking and make sense of others' thinking</li> <li>✓ Make connections with the math learned in the classroom to the community and the world</li> <li>✓ Use think-alouds to share how mistakes were identified and corrected as part of understanding concepts</li> </ul>





### Assessment Considerations

Effective assessment for productive disposition focuses on how students approach challenging tasks, reflect on their learning, and perceive themselves as capable mathematical learners. These assessments should be embedded in daily instruction and aligned with grade-level mathematics standards and the Standards for Mathematical Practice, particularly perseverance and sense-making (SMP 1). Common assessment approaches include student reflection journals, math surveys or self-assessments, teacher observations of persistence and collaboration, goal-setting reflections, and discussion prompts or exit tickets that capture how students overcome challenges. Together, these tools provide insight into students' mathematical mindset and support the development of confidence, resilience, and a positive disposition toward learning mathematics.

### Progressions

#### Young Learners: Concrete Foundations

- ✓ Develop confidence through success with manipulatives and games
- ✓ See math as exploration and sense-making

**Example:** "Find different ways to make 10 and explain your thinking." (K.OA)

#### Elementary: Expanding Relationships

- ✓ Engage in multi-step problems with support
- ✓ Develop positive attitudes toward challenge

**Example:** "Solve a multi-step word problem and explain why you used your selected strategy." (5.NBT)

#### Middle School: Abstract Connections

- ✓ Persist through complex problems and revise thinking
- ✓ Reflect on mistakes

**Example:** "Analyze the error and explain how to solve the problem correctly." (8.EE)

#### High School: Structural Understanding

- ✓ Engage independently with complex, open-ended tasks
- ✓ Develop ownership of mathematical reasoning and persistence

**Example:** "Construct a quadratic model to represent a real-world scenario and justify your assumptions and revisions." (F-LE)

## Discourse

### Discourse

Mathematical discourse is the purposeful communication of mathematical ideas, including the tools and practices that make thinking visible. Discourse can take place verbally, visually, or in written formats. It involves students articulating their reasoning, engaging with peers' ideas, and collaboratively constructing mathematical understanding. Mathematical discourse promotes students' understanding of mathematical concepts and procedures and can make a positive impact on productive disposition.<sup>23</sup> Through discussion and justification of mathematical ideas, teachers gain insight into students' conceptual understanding, problem-solving strategies, and misconceptions, enabling timely and targeted instructional adjustments.<sup>24</sup>



### Why It Matters

Mathematical discourse is essential because it is the primary means through which students develop, test, and refine their understanding of mathematics. In everyday life, individuals must communicate quantitative ideas clearly, justify decisions, and interpret the reasoning of others. Examples include explaining a financial choice, discussing data presented in the media, or collaborating to solve a problem at work. Through purposeful discussion and writing, students learn to articulate their thinking, question ideas, and make sense of multiple approaches, deepening both understanding and reasoning. This process not only strengthens conceptual understanding but also builds the ability to evaluate arguments and communicate with clarity and precision. When classrooms prioritize discourse, they create equitable environments where all students' ideas are valued, and learning is driven by sense-making rather than speed or memorization. Errors become opportunities for growth, and understanding is constructed collaboratively through dialogue and reflection. Ultimately, discourse positions mathematics as something students actively do, share, and apply in real-world contexts.

- ✓ Deepens conceptual understanding through articulation and revision of thinking
- ✓ Builds reasoning and problem solving through explanation and argumentation
- ✓ Improves transfer of learning across contexts and representations
- ✓ Supports equity by making thinking visible and accessible to all learners
- ✓ Strengthens procedural fluency through explanation, not memorization alone



### Instructional Practices and Strategies<sup>25</sup>

#### Aligned with Evidence

- ✓ Structured math talk routines (think-pair-share, number talks, debate protocols)
- ✓ Explicit teaching and modeling of mathematical vocabulary
- ✓ Teacher facilitation that prioritizes student-to-student discourse over teacher explanation
- ✓ Formative assessment through listening and analyzing student reasoning
- ✓ Norm-setting for respectful disagreement and revision of thinking

#### Not Aligned with Evidence

- ✗ Heavy reliance on teacher-led explanation with limited student talk
- ✗ Tasks that have only one correct method with no opportunity for justification
- ✗ Discourse limited to “answer sharing” rather than reasoning explanation
- ✗ Correction of language before meaning is developed (focus on grammar over reasoning)
- ✗ Group work without accountability for mathematical talk
- ✗ Limited wait time after posing questions

### Specialized Learning Considerations

#### Multilingual Learners

- ✓ Use visual models, gestures, and structured talk frames
- ✓ Provide bilingual glossaries and cognate support
- ✓ Allow translanguageing to develop conceptual understanding
- ✓ Pair oral reasoning with diagrams and manipulatives

#### Students with IEPs/504 Plans

- ✓ Provide sentence starters and guided discourse scaffolds
- ✓ Reduce linguistic load while maintaining cognitive demand
- ✓ Offer alternative expression modes (oral, visual, recorded responses)
- ✓ Pre-teach vocabulary and concepts before discourse tasks

#### Gifted & Advanced Learners

- ✓ Extend discourse into justification of multiple strategies or proofs
- ✓ Require critique of efficiency and generalization
- ✓ Encourage student-led facilitation of math discussions
- ✓ Introduce “always/sometimes/never” justification tasks

#### Students with Dyscalculia

- ✓ Emphasize verbal reasoning supported by visual representations
- ✓ Reduce memory load through manipulatives and reference tools
- ✓ Use structured talk routines to externalize thinking
- ✓ Prioritize conceptual explanation over procedural recall

## Discourse

SMPs	Teacher Actions	Student Actions
MP.3 MP.6	<ul style="list-style-type: none"> <li>✓ Explicitly teach what math discourse looks and sounds like</li> <li>✓ Provide adequate wait time before calling on students</li> <li>✓ Use think-pair-share, turn and talk, or small group structures before whole class discussion</li> <li>✓ Plan sequenced tasks to encourage comparison of strategies</li> <li>✓ Ask probing questions such as: Why does that work? How do you know? What would happen if...?</li> <li>✓ Ask students to share their thinking visually, verbally, or in writing</li> <li>✓ Connect students' contributions to lesson goals or concepts</li> <li>✓ Model precise mathematical language in authentic contexts</li> <li>✓ Ask students to reflect on how their thinking changed</li> <li>✓ Listen carefully to student talk to inform instructional decisions</li> </ul>	<ul style="list-style-type: none"> <li>✓ Justify an answer using reasoning or evidence</li> <li>✓ Connect methods to a model, design, or equation</li> <li>✓ Ask clarifying questions about vocabulary, strategies, a peer's reasoning, etc.</li> <li>✓ Rephrase a strategy to check for understanding</li> <li>✓ Agree, disagree, or expand on a peer's idea/thinking</li> <li>✓ Use appropriate mathematic vocabulary</li> <li>✓ Analyze, critique, evaluate, and argue to defend or define reasoning, errors, misconceptions, etc.</li> <li>✓ Reflect, revise, and summarize thinking</li> </ul>





### Assessment Considerations

Discourse assessment is embedded within meaningful tasks and supports deeper conceptual understanding, multiple solution pathways, equity, and students' capacity to construct and critique mathematical arguments using evidence. Formative assessments of mathematical discourse are ongoing and include strategies such as observing student discussions, collecting exit tickets with explanations, using turn-and-talks, and analyzing annotated work samples to inform instruction. Summative assessments evaluate cumulative understanding through tasks like written mathematical arguments, performance-based problem solving with explanations, discourse portfolios, or oral defenses. Together, these approaches ensure that assessment measures not only procedural accuracy but also the quality, clarity, and depth of students' mathematical thinking and communication.

### Progressions

#### Young Learners: Concrete Foundations

- ✓ Orally explain thinking using everyday language and emerging math vocabulary
- ✓ Use manipulatives and drawings to show reasoning
- ✓ Participate in structured talk

**Example:** "Explain how you know 8 is more than 5 using objects or drawings." (K.CC)

#### Elementary: Expanding Relationships

- ✓ Use sentence frames ("I noticed...", "I agree because...")
- ✓ Begin justifying strategies using operations and models
- ✓ Compare and critique solution methods

**Example:** "Compare two strategies for  $347 + 129$  and explain which is more efficient and why." (4.NBT)

#### Middle School: Abstract Connections

- ✓ Use increasingly precise mathematical language and reasoning
- ✓ Justify and critique strategies using properties, representations, and relationships among quantities
- ✓ Build on, question, and revise ideas

**Example:** "Compare two different methods for solving a multi-step ratio and explain which method is more efficient or reliable." (6.RP)

#### High School: Structural Understanding

- ✓ Construct formal proofs and multi-step reasoning
- ✓ Defend assumptions and model choices using precise notation
- ✓ Engage in academic discourse using discipline-specific language

**Example:** "Prove or disprove that the product of two irrational numbers is always irrational." (N-RN)

## Connecting Content, Practices, and Components of Numeracy

Connecting content, the SMPs, and the Six Components of Numeracy is essential for coherent mathematics instruction. The Illinois Learning Standards define what students should know and be able to do by the end of each grade level, while the SMPs describe how students engage with the learning. Through sustained engagement with both, students develop numeracy skills including conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, productive disposition, and discourse skills. These elements are not separate, nor are they sequential; they work together within and across lessons to support meaningful and connected learning. Figure 5 illustrates how these components interact to support student understanding over time.

Teachers intentionally weave together content and practices through tasks, discussions, and assessments. Students engage actively, make connections, and build understanding over time.



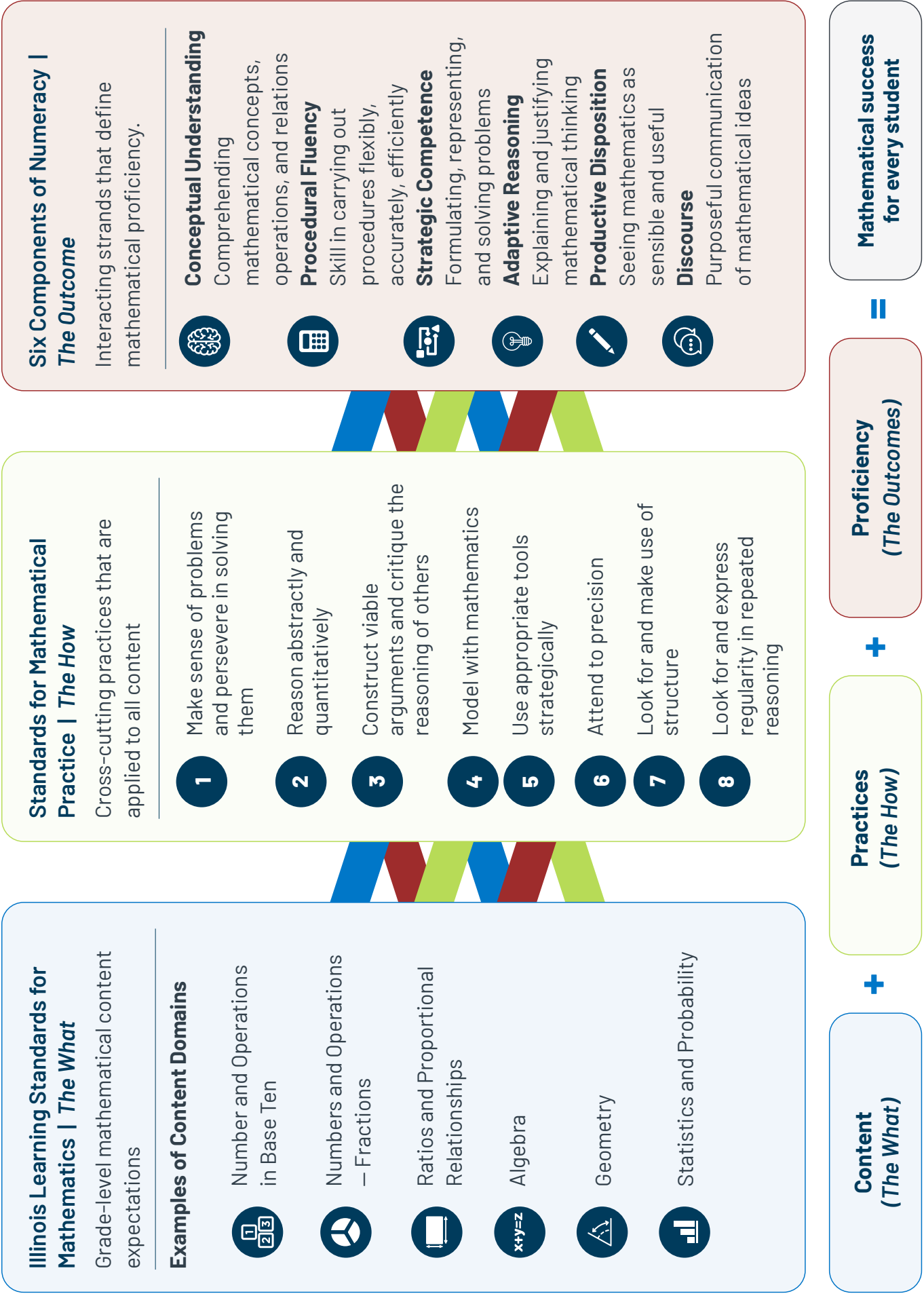


Figure 5: Relationships Among Content, Practices, and Components

## Instructional Considerations

### Considerations for All Learners

Ensuring that every student develops numeracy skills requires a commitment to access, equity, and differentiation. Evidence shows that when instruction is designed to meet diverse learning needs, all students are more likely to engage meaningfully with mathematics and to see themselves as capable problem solvers.<sup>26</sup> This section emphasizes that numeracy is not reserved for some students—it is a right for all learners.

### Student Engagement and Mindset

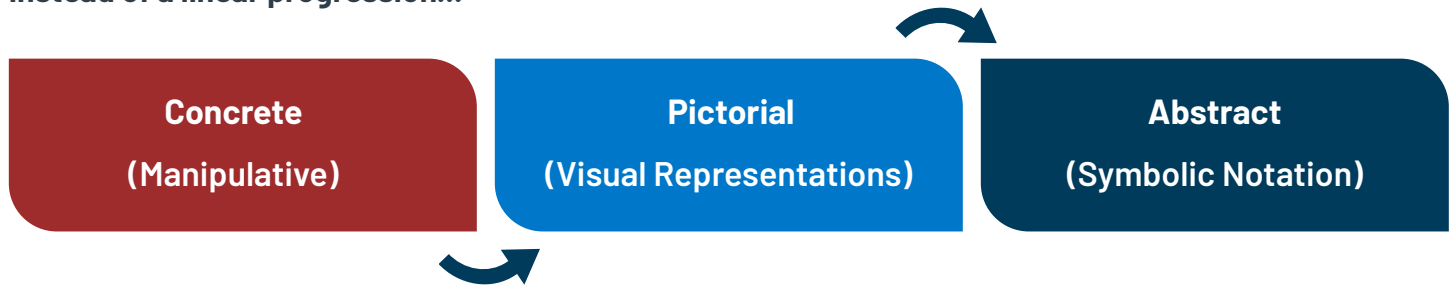
Engagement, curiosity, and mindset shape whether students persist with challenging problems, see value in mathematics, and develop confidence in their ability to learn. High-quality mathematics instruction should deliberately cultivate positive mathematical mindsets and mathematical curiosity by:

- **Building relevance and wonder:** Connecting mathematics to students' interests, cultural contexts, and community issues increases engagement and demonstrates the usefulness of numeracy beyond the classroom. Presenting open-ended problems or puzzles sparks curiosity and invites students to ask "what if" questions.
- **Normalizing productive struggle:** Framing mistakes as opportunities for learning helps students see challenge as part of the process, not as a sign of inability. Curiosity grows when students feel safe to explore and experiment and are provided the time to do so.
- **Encouraging voice, agency, and discourse:** Providing opportunities for students to explain their reasoning, share strategies, ask questions, and engage in rich mathematical discussions fosters ownership of learning. The intentional use of precise mathematical vocabulary by both teachers and students is essential for developing clear communication and deep conceptual understanding.
- **Strengthening belonging:** Classrooms that emphasize collaboration, respect, and inclusion counteract stereotypes and help students feel they belong in mathematics.
- **Low-floor, high-ceiling tasks:** Instruction should also include low-floor, high-ceiling tasks, which provide multiple entry points for learners of varying abilities while allowing all students opportunities to progress and deepen their understanding.<sup>27</sup> Tasks should be authentic and, while often messy, reflect real-world contexts, current interests, or future possibilities. Authentic tasks challenge students to reason, model, problem solve, and engage in meaningful discourse, which fosters and promotes curiosity through exploration.
- **Valuing diverse ways of knowing:** Inviting multiple approaches and representations allows students to see their thinking as valid and important and encourages exploration of alternative solutions. Students in all grades benefit from the use of manipulatives in transitioning from concrete to abstract representations. Students need to have ongoing access to manipulatives over extended periods of time (not just for a single lesson or unit) to support sustained development of conceptual understanding. Learning progressions are not linear; rather, they are integrated and overlapping as students make transitions between levels of understanding. Teachers must guide students in making connections between manipulatives, prior knowledge, and the current concept to promote conceptual understanding.<sup>28</sup> See Figures 7 through 9 on pages 54 and 55 depicting examples of CPA progressions.



Engagement is not only about motivation but also about creating conditions for active participation, exploration of curiosity, and collaborative discourse. When students are engaged in asking questions, investigating patterns, and discussing, defending, and critiquing mathematical ideas, they may make connections to other subjects, civic life, and future careers. Fostering mathematical confidence, curiosity, persistence, and discourse ensures that students leave school with the skills needed to navigate the 21st century.<sup>29</sup>

**Instead of a linear progression...**



**...Plan for an integrated and overlapping progression in which students may move flexibly between representations:**

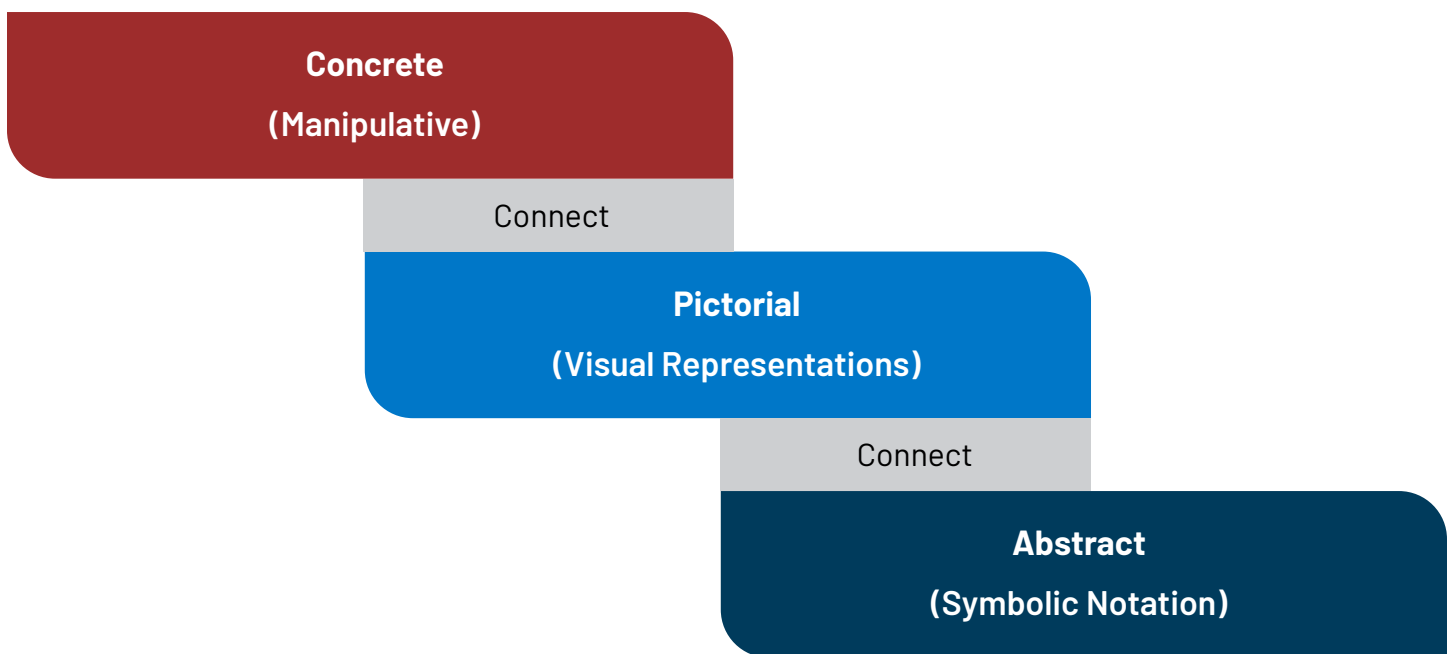
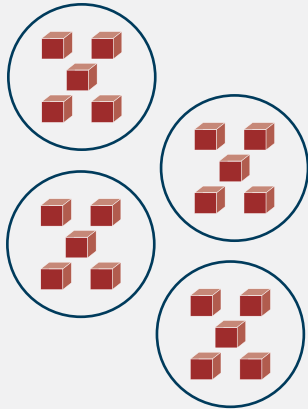


Figure 6: Integration of CPA Progressions

## Problem: $4 \times 5 = 20$

Concrete:



Pictorial

**5**

<b>4</b>				

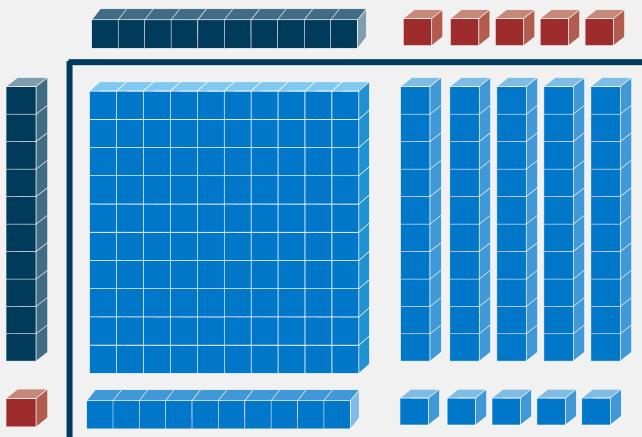
Abstract:

$$4 \times 5 = 20$$

Figure 7: CPA Progressions of Single-Digit Multiplication

## Problem: $15 \times 11$

Concrete:



Pictorial:

	<b>10</b>	<b>5</b>
<b>10</b>	<b>100</b>	<b>50</b>
<b>1</b>	<b>10</b>	<b>5</b>

$$100 + 50 + 10 + 5 = 165$$

Abstract:

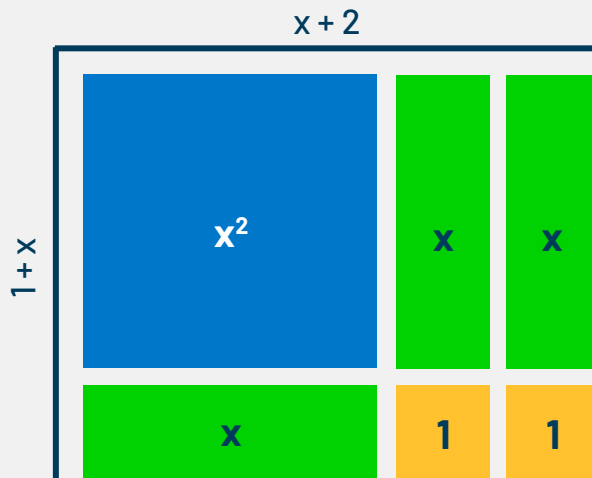
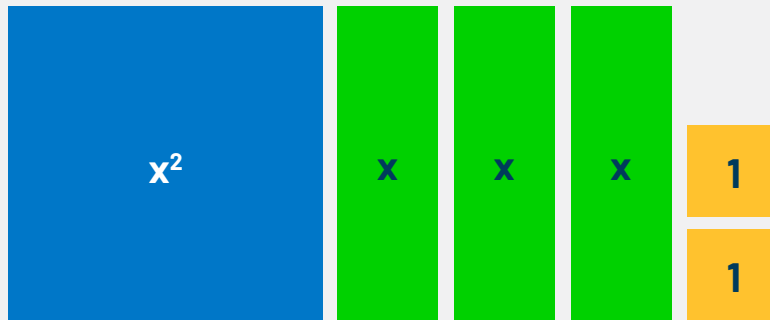
$$\begin{array}{r} 15 \\ \times 11 \\ \hline 15 \\ + 150 \\ \hline 165 \end{array}$$

Figure 8: CPA Progressions of Multi-Digit Multiplication

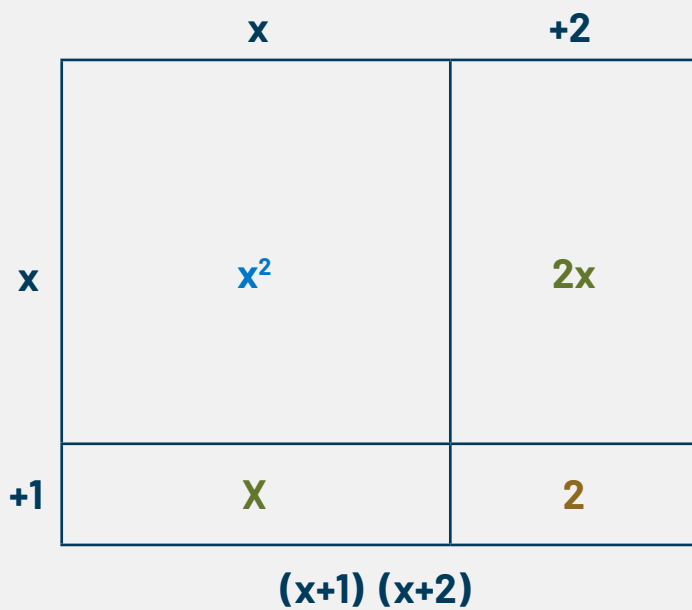


# Factor the polynomial: $x^2+3x+2$

Concrete:



Pictorial:



Abstract:

$$\begin{array}{r}
 x^2+3x+2 \\
 (x+1)(x+2)
 \end{array}
 \qquad
 \begin{array}{r}
 2 \\
 \hline
 1 \quad | \quad 2 \\
 \hline
 \end{array}$$

Figure 9: CPA Progressions of Factoring Polynomials

	Teacher Actions	Student Actions
<b>Introduction of a Concept</b>	Presents a real-world problem (e.g., sharing snacks, calculating discounts, designing a garden) and models the concept using concrete materials, visual representations, and symbolic notation. Uses Universal Design for Learning (UDL) principles to provide multiple entry points (visual, verbal, kinesthetic, digital). <i>View additional information regarding UDL on page 70.</i>	Engages with the concept through manipulatives (concrete), diagrams or drawings (pictorial), and numerical symbols or equations (abstract).
<b>Problem Solving</b>	Facilitates a group or individual problem-solving activity. Encourages students to try multiple strategies, such as drawing diagrams or models, using manipulatives, applying mental math or algorithms, and exploring patterns and generalizations. Supports collaboration and discussion while scaffolding for diverse learners.	Works with partners or independently to explore solutions using concrete, representational, and abstract methods. Tries multiple strategies, justifies choices, and demonstrates flexibility in finding solutions.
<b>Consolidating Learning</b>	Collaboratively creates anchor charts, graphic organizers, or digital representations with student input. Highlights key strategies, patterns, and connections across representations.	Contributes ideas, examples, manipulatives, diagrams, and equations to the shared reference, reinforcing understanding across CPA stages.
<b>Check for Understanding</b>	Uses formative assessments, such as exit tickets, quick sketches, oral explanations, or short performance tasks, to evaluate understanding across CPA stages.	Responds with hands-on materials, drawings, or symbolic work. Demonstrates understanding and identifies areas needing clarification.
<b>Deepening Understanding</b>	Challenges students to explain reasoning, compare multiple approaches, justify solutions, and connect ideas across CPA levels and conceptual, procedural, and application dimensions of the standards as written. Promotes rich discourse and reflection on mathematical thinking.	Explains thought processes, critiques peer strategies, applies reasoning to new problems, and demonstrates mastery through concrete, representational, or abstract methods.
<b>Extending &amp; Enrichment</b>	Provides parallel tasks that allow all students to engage meaningfully while offering opportunities to extend reasoning or complexity for advanced learners. Connects tasks to real-world contexts and future applications.	Engages in tasks at their level of readiness, explores advanced or creative strategies, and applies concepts in novel or complex situations.

Figure 10: Actions to Build Mathematical Confidence



## Differentiation

Effective differentiation in mathematics instruction recognizes the assets students bring to the classroom, including their cultural and linguistic resources, while also addressing gaps that may arise from systemic inequities. Instructional planning should consider all learners, not just the “average” student, ensuring that outliers, both students who need additional support and those who require enrichment, have meaningful access and challenge.<sup>30</sup>

Universal Design for Learning (UDL) instruction is designed to provide multiple means of engagement, representation, and expression. In practice, UDL encourages teachers to present information through diverse modalities, such as visuals, text, audio, manipulatives, or interactive simulations, so students with varying learning preferences can grasp concepts.<sup>31</sup> It also allows students to express their learning in multiple ways, including writing, speaking, drawing, or using digital tools, and provides varied opportunities for engagement through choice, real-world applications, collaboration, and appropriately challenging tasks. This approach ensures that all students can access, participate in, and demonstrate understanding of grade-level content, regardless of learning profile or background. *View additional information regarding UDL on page 70.*

Low-floor, high-ceiling tasks are intentionally designed to provide an accessible entry point for all students (low floor), while offering opportunities for extended reasoning and challenge (high ceiling). Such tasks allow students to engage at their level, explore multiple solution strategies, and deepen conceptual understanding.<sup>32</sup>

Additional differentiation strategies include:

- Offering varied entry points to mathematical concepts through visual, verbal, kinesthetic, or digital modalities
- Providing scaffolds and extensions such as intervention, remediation, or enrichment to learning so that tasks remain accessible yet rigorous for all learners
- Encouraging multiple-solution strategies and representations to honor diverse ways of reasoning
- Embedding opportunities for student choice and voice, which builds ownership of learning and affirms students’ mathematical identities

### For example:

Students are asked to design a rectangular 24-square-meter school garden, using whole-number side lengths. They must determine all possible dimensions and explain their reasoning. Students who are developing basic multiplication skills can use grid paper or manipulatives to count squares and explore simple length  $\times$  width combinations, such as  $4 \times 6$  or  $3 \times 8$ , providing a low-floor entry point. Advanced students can extend the task by generalizing patterns, calculating perimeter and fencing costs, considering fractional or mixed-number dimensions for irregular plots, or modeling the problem algebraically. They might also opt for additional variables, such as sunlight exposure or plant spacing, creating a high-ceiling challenge that encourages rich mathematical reasoning and multiple-solution strategies.

Equity in differentiation requires more than adjusting instruction within classrooms. It also involves ensuring that all students, regardless of background, prior achievement, or learning profile, have access to grade-level content, advanced coursework, and enrichment opportunities. Systemic barriers that track or limit students must be dismantled to create a truly equitable numeracy experience across Illinois schools.<sup>33</sup>

By intentionally planning for outliers and integrating UDL principles with low-floor and high-ceiling tasks, educators can meet students where they are, provide rigorous pathways forward, and ensure that all learners can engage meaningfully in mathematics.

## Scaffolding

Scaffolding is an effective teaching method whereby teachers provide targeted supports for students to learn new concepts by building on prior knowledge. Scaffolding is different from differentiation. Scaffolding provides students with supports they may need to complete a task, while differentiation involves creating different tasks for different students based upon student needs. Effective instruction balances support and independence. Scaffolds should be deliberately removed as students develop competence, ensuring they move confidently through concrete, representational, and abstract stages while building mathematical resilience and problem-solving skills.<sup>34</sup>

### ✓ Do

- ✓ Personalize scaffolds based upon students' zone of proximal development
- ✓ Guide student thinking through correcting, modeling, cluing, or prompting
- ✓ Gradually fade supports as students become more independent
- ✓ Attach scaffolds to grade level and standards-aligned tasks

### ✗ Don't

- ✗ Provide all students with the same supports
- ✗ Provide students with answers
- ✗ Remove supports too soon or keep them indefinitely
- ✗ Use remedial work in place of rigorous learning tasks



	Definition	Visual	Verbal	Written
	<p><b>Self-Scaffolding</b></p> <p>Students independently draw on prior knowledge, strategies, and reasoning to plan, monitor, and evaluate their approach to a task. Educators observe, give space for thinking, and intervene only when necessary.</p>	<p>Students independently create visual reminders or organizers (e.g., number lines, arrays, coordinate grids, diagrams, geometric models) that help them consolidate learning.</p>	<p>Students generate questions to support metacognition or next steps (e.g., “What do I know about this type of problem?” “How else could I represent this?” “Does my answer make sense?”).</p>	<p>Students independently create success criteria, checklists, or self-evaluation tools to monitor their work. (e.g., multi-step word problems, fraction comparisons).</p>
	<p><b>Prompting</b></p> <p>Educators provide prompts that encourage students to apply what they already know without directing them toward a specific method. Prompts nudge students toward productive strategies and independence.</p>	<p>Educators encourage students to identify classroom visuals (e.g., fact families, multiplication charts, geometric vocabulary walls, fraction models) that may support decision-making.</p>	<p>Educators offer reflective or planning questions such as: “What strategy could you try first?” “Where have you seen a problem like this?” “Which representation might help you understand this better?”</p>	<p>Educators encourage students to revisit written scaffolds they have developed previously (e.g., steps for solving equations, writing frames for explaining reasoning, structures for math arguments).</p>
	<p><b>Cueing</b></p> <p>Educators provide targeted hints that point students toward key information or conceptual connections while still requiring them to think and problem-solve.</p>	<p>Visual cues highlight essential concepts or representations (e.g., equal groups, symmetry, place value shifts, slope triangles, partitioned fraction bars) to support understanding.</p>	<p>Educators offer verbal clues connecting to strategies previously used successfully (e.g., “Think about how the numbers are related,” “Check the units,” “What operation fits this context?”)</p>	<p>Educators provide partially completed examples, sentence starters, word banks that help students engage with a task independently (e.g., labeled number lines, incomplete tables, partially filled area models)</p>

Student is least independent

Definition	Visual	Verbal	Written
<p><b>Modeling</b></p> <p>Educators model a process, strategy, or representation so students can observe expert thinking. Students are expected to apply the modeled strategy shortly afterward.</p>	<p>Educators demonstrate the use of manipulatives, diagrams, or models for students to reference when completing their own work.</p>	<p>Educators model high-quality discourse or reasoning (e.g., "If I were justifying my solution, I might start by saying...").</p>	<p>Educators offer worked examples as references, ensuring students apply, but not copy, the structure and reasoning shown.</p>
<p><b>Correcting</b></p> <p>Educators identify errors or misconceptions and guide students in understanding the correction. Correcting should be used sparingly and only when conceptual clarity is necessary for learning to continue.</p>	<p>Educators annotate student work or highlight representations that need revision without completing the work for the student.</p>	<p>Educators verbally explain misconceptions (e.g., "This step doesn't follow because... Let's examine the relationship between these quantities again.").</p>	<p>Educators provide accurate models, spellings, or symbolic notation for students to reference or rewrite when appropriate.</p>

Figure 11: Scaffolding Framework<sup>35</sup>



## Addressing Misconceptions

A critical component of maintaining a coherent learning continuum is the intentional identification and instructional use of student misconceptions. Misconceptions should not be viewed simply as errors to correct but as evidence of students' current reasoning and partial understandings. Effective mathematics instruction requires teachers to anticipate likely misconceptions, elicit student thinking through purposeful questioning and formative assessment, and use that information to make real-time instructional decisions that refine and extend understanding.<sup>36</sup>

Rather than immediately correcting errors, teachers should create opportunities for students to examine misconceptions through discussion, comparison of strategies, and engagement with multiple representations. This approach positions misconceptions as productive entry points for deeper learning and strengthens students' conceptual understanding over time.

In some cases, misconceptions can be unintentionally reinforced through instruction that emphasizes procedural "tricks" or shortcuts without conceptual grounding. For example, teaching students to "move the decimal" when multiplying by powers of 10 can obscure understanding of place value, and suggesting that an inequality symbol "points to the direction to shade on the number line" can lead to errors when graphing or interpreting solutions. While these shortcuts may produce correct answers in limited situations, they do not build number sense or transferable understanding and can break down in more complex contexts.

If misconceptions are not addressed with intention, they can solidify and disrupt students' ability to access future content. For example, when first learning fractions, students may believe that a larger denominator indicates a larger fraction (e.g., thinking  $1/8 > 1/4$ ). Addressing this misconception requires more than correction; it involves engaging students with visual models, number lines, and reasoning tasks that help them reconcile the relationship between unit size and quantity. This type of targeted instructional response supports stronger understanding of fraction magnitude, which is foundational for later work with equivalence, operations with rational numbers, and proportional reasoning.<sup>37</sup>

## Collaboration

Collaboration is essential for maintaining a cohesive continuum of learning. Educators need structured opportunities to examine standards and progressions across grade levels, identify where students may struggle, and design instructional supports. Families also play an important role in reinforcing numeracy development across a child's lifetime, especially in early years, and should be viewed as educational partners by educators and administrators.<sup>38</sup>

Collaboration and communication between general education and special education staff are crucial to supporting numeracy growth. Successful inclusive instruction requires structured, ongoing communication about what is being taught, how it is taught, and the strategies and accommodations being applied, as well as considerations for advanced learners. When general education teachers bring strong content knowledge and special education teachers contribute expertise in individualized supports and executive functioning, a co-planning model can maximize access for all students.

Shared ownership includes:

- Co-planning for differentiation and accessibility in advance of instruction
- Structured communication protocols between general education teachers, special education teachers, and paraeducators, including discussion of goals, strategies, assessments, classroom observations, and data
- Intentional scaffolding of mathematical discourse to ensure all students can participate, build understanding, and develop a positive mathematical identity

Additionally, integrated learning opportunities for students can increase numeracy learning. Mathematics teachers are not solely responsible for numeracy instruction as all core content areas can support numeracy skills. When students see that mathematical skills can be applied across all content areas, they are more likely to realize that mathematics is necessary and relevant in their everyday lives rather than just a skill they need to pass a test or learn to prepare for the next grade level. Integrated curriculums have been shown to instill a positive mindset among students as well as increase student motivation and achievement.<sup>39</sup> Collaboration among grade-level teachers can promote more opportunities for integrated learning of all subjects to better support student achievement, especially in areas in which there is already overlap or close alignment of skills or practice standards.

## Specialized Learning Considerations

Students with specialized learning needs, including those with disabilities, multilingual learners, and students requiring additional supports, as well as those who are gifted and talented all benefit from evidence-based practices grounded in the principles of UDL.<sup>40</sup>

Key considerations include:

- **Accessibility:** Tasks and assessments should be designed to reduce barriers and allow all students to demonstrate mathematical understanding. This includes providing multiple means of representation (visual models, manipulatives, etc.), offering options for how students express their thinking (verbally, in writing, or visual representations) and ensuring tasks maintain mathematical rigor while removing unnecessary linguistic or procedural barriers. Educators should audit tasks and activities during the planning process to identify potential barriers to learning and proactively plan supports that increase access to meeting the learning targets.
- **Scaffolding and Supports:** Scaffolds include using tools such as manipulatives, technology, visual aids, and structured peer collaboration. Effective scaffolding involves modeling strategies, cueing conceptual connections, and prompting students to apply prior knowledge to solve problems while maintaining high cognitive demand. Worked examples, structured discourse routines, and intentional use of representations further support access to complex material. Educators should intentionally fade supports over time based on student understanding to promote independence and transfer.
- **Flexible Pacing and Pathways:** It's important to recognize that students may progress through numeracy development at different rates and require personalized supports. This may include using formative assessments to adjust instruction in real time, offering varied entry points to tasks, and providing opportunities for extension or additional practice based on student need. Options for flexibility such as choice boards or parallel tasks should be included in lessons to engage students at an appropriate instructional level.



- **Strength-Based Approaches:** Teachers must identify and build on students' assets. Instruction should intentionally leverage students' prior knowledge, cultural and linguistic resources, and problem-solving strategies to position students as capable learners and promote positive mathematics identities. Teachers can incorporate opportunities for students to share and compare strategies, highlighting diverse ways of thinking as an asset to learning.
- **Collaboration:** General educators, special educators, and support staff must work together to deliver coherent and aligned instruction. Collaborative planning, co-teaching models, and data-driven instruction support consistency in expectations and teaching methods, leading to more inclusive and effective environments. Establishing regular structures for collaboration such as common planning times or data meetings helps ensure support is coordinated and responsive to student needs.

When classrooms implement these considerations, students with specialized needs, including those who require intervention or enrichment, not only access the same numeracy content as their peers but also engage in meaningful problem solving, reasoning, and communication. By adopting this approach, Illinois schools can ensure all students, regardless of ability, background, or need, have equitable opportunities to thrive mathematically.

## Considerations for Learners with Specialized Education Needs

Professional learning grounded in evidence-based practices, such as those outlined in [Institute of Education Sciences Practice Guides](#), is essential for equipping educators with strategies that are effective for students with learning disabilities and beneficial for all learners. Learners with specialized learning needs must be exposed to grade level content as well as receive instruction that supports the Six Components of Numeracy. Students should be supported with appropriate learning accommodations. Collaborative efforts between general education teachers and special education teachers must be prioritized to ensure learners with specialized education needs are provided with equitable access to rigorous mathematics instruction. Co-teaching, when possible, is the ideal solution to ensure this collaboration as well as to account for the common differences in educator preparation among special education and general education teachers.<sup>41</sup>

Early screening is important to identify students at risk and those with specialized education needs who may need evidence-based supports. Identification takes place via screening and assessment, and learning supports for individuals are determined through Individualized Education Programs. Explicit and systematic approaches are especially important for specialized education needs.

## Considerations for Learners with Dyscalculia

Dyscalculia is a learning disability that affects how students learn, understand, and retain mathematical concepts.<sup>42</sup> It is a specific learning disorder that impacts number sense, magnitude, and the ability to make sense of numerical relationships.<sup>43</sup>

Students with dyscalculia may experience difficulty with:

- Understanding number relationships and magnitude
- Counting and number sense development
- Math fact retrieval and fluency
- Multistep computation and procedures
- Interpreting math symbols
- Understanding place value and number structure



Effective instruction for students with dyscalculia requires intentional design that prioritizes clarity, structure, and conceptual understanding.<sup>44</sup> Key considerations include:

- Use clear, direct language and break instruction into manageable steps. Check for understanding throughout the process and create time and space for students to ask for clarification.
- Use visual supports, checklists, and think-alouds to make procedures transparent and support independent problem solving over time.
- Consistently use and provide access to concrete and visual supports. Manipulatives, number lines, visual models, and reference tools help students develop number sense and should remain available across grade levels and tasks when needed.
- Use visual step-by-step modeling and guided practice. Structured checklists and worked examples help students retain processes, increasing independence over time.
- Point out patterns and connections in mathematics for students. Students with dyscalculia benefit from explicit support in recognizing structure in mathematics (e.g., place value patterns, algebraic structure, or relationships in operations). Visual cues and schema-based learning can assist students in identifying structures.
- Support students' emotional well-being and confidence in mathematics. Students with dyscalculia may experience increased math anxiety, frustration, or avoidance due to repeated struggle. Instruction should intentionally normalize mistakes, provide frequent opportunities for success, and create a supportive environment that builds confidence and reduces fear of failure in mathematics.

### Considerations for Multilingual Learners

To support multilingual learners (MLs) as they become competent and confident problem solvers and contributors of mathematics knowledge while engaging in a mathematical discourse community, Celedón-Pattichis & Ramirez suggest the following guiding principles for teaching mathematics:

<b>Challenging mathematical tasks</b>	<b>Linguistically sensitive social environments</b>	<b>Support for learning English while learning mathematics</b>	<b>Mathematical tools and modeling as resources</b>	<b>Cultural and linguistic differences as intellectual resources</b>
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Effective teachers of MLs address these principles by using the following practices and actions in their classrooms<sup>45</sup>:

- At the beginning stages of ML development, limit the use of various academic terms for calculations in word problems. Wait until MLs have shown mastery before introducing new terms.
- Use challenging problems and tasks and assess MLs' prior knowledge of the problem's background and context. This includes culturally relevant problems or tasks, planning the use of multiple tools, representations, and models, and focusing on what mathematical and everyday language demands of the problem or task.
- Model making sense of written instructions prior to using these instructions on exams so students are not seeing these instructions for the first time on an exam.



- Form groups based on the cognitive demand of the task or problem and awareness of how students may use their native language to process and reason their way to a solution, particularly as the complexity of the problem increases. Try to avoid calling attention to a student who appears to need additional support.
- Sequence problems and tasks carefully to allow MLs to develop competence with mathematics while developing proficiency with language, both mathematical and English.
- Facilitate and model mathematical discourse (e.g., provide sentence starters, visuals, use topics that are familiar, provide models of how to respond or ask questions) in class so that students can develop conceptual understanding and procedural fluency while increasing their language proficiency.
- Provide support through multiple modalities (e.g., visual, auditory, native language) to address complexity of language demands in problems and tasks.
- Allow processing time while recognizing the mental requirements students have with speaking, reading, listening, and writing in a new language.
- Provide opportunities for MLs to read and write about their learning of mathematics, receive feedback that purposefully addresses mathematical language development, and allow students to revise their writing to support students in their learning of language. Include opportunities for problems and tasks with rich mathematical contexts and sophisticated language to help students advance their language development and mathematical learning.
- Provide space in the classroom where mathematical vocabulary and meaning are visible, referred to often during instruction, and frequently used in students' oral and written communications.
- Allow for the use of multiple tools and representations to enhance mathematical discourse and understanding. Intentionally teach the use of these tools to MLs. Based on background and/or prior experience, students may be unfamiliar with calculators, math apps, or other manipulatives. Provide students the opportunity to practice with the tools after teaching their use.
- Make visual connections to mathematical models and representations found in the classroom.
- Call attention to the peculiarities between meanings of words (e.g., words with multiple meanings such as yard, area, times, and others, as well as homophones like eight and ate) used in both everyday language and mathematical language.
- Meet the needs of students by listening to what they have to say while providing supporting guidance and mentoring and advocating for their rights as students and individuals.
- Learn about students' culture and community, language, dialects, and ways of knowing and understanding to enhance the teacher-student experience. Be cognizant of cultural differences in mathematical representations and notations, writing, and problem solving.
- Model acceptance and interest in students' language and culture to show MLs that they are valued for what they have to offer the class. This can have a positive effect on students' learning and identity.
- Communicate and collaborate regularly with your school or district's ESL or bilingual teacher(s).
- Identify the background knowledge a student may need to solve a word problem. Unfamiliarity with the topic of the word problem may be a barrier to the student demonstrating their understanding of the mathematical concepts being assessed. For example, in a mathematics problem about receiving a coupon for a free t-shirt at a football game, the student may be distracted trying to figure out what a coupon is, and whether football and soccer are the same sport.

## Considerations for Advanced Learners

Advanced learners are present in every district and across all levels of socioeconomic status, and educators must be equipped with the skills and knowledge to support them. The following strategies and best practices can be used to create a supportive, equitable learning environment for advanced learners.

Strategy	Definition	Additional Information
<b>Acceleration</b>	Moving through the curriculum at a pace that challenges students appropriately, preventing boredom and stagnation	Examples of acceleration include skipping a grade level, early entrance into kindergarten or college, and entrance into dual credit or Advanced Placement courses.
<b>Curriculum Compacting</b>	Modifying the regular curriculum to skip previously mastered learned material to focus on more challenging numeracy tasks	Educators must begin curriculum compacting by identifying the intended instructional outcomes of a unit. A pre-assessment can then be used to determine which skills students have previously mastered and which skills must still be developed. Based upon results, educators will then provide acceleration or enrichment.
<b>Grouping</b>	Placing students with similar ability levels together for more advanced instruction and rapid growth	Grouping should be flexible to allow for variation and avoid tracking with the goal of matching student readiness with instruction.
<b>Community Partnerships</b>	Collaborating with local businesses, museums, universities, community members, etc.	External partners can provide resources and supports and offer students opportunities for real-world applications.
<b>Enrichment</b>	Extending learning experiences beyond the standard curriculum and invite students to explore numeracy more deeply	Deepening knowledge occurs when students engage with current content through complex problem solving, investigation, multiple-solution strategies, etc., strengthening conceptual understanding and the ability to apply learning across contexts. Enrichment can occur in the classroom or through pull-out programs.

Figure 12: Considerations for Advanced Learners<sup>46</sup>



## Multi-Tiered Systems of Support

A strong Multi-Tiered System of Support (MTSS) is foundational to ensuring that every student in Illinois develops numeracy. This system is designed to enhance the core learning environment, identify struggling students early and those who would benefit from extension; offering timely interventions and enrichment opportunities that encompass various aspects of a child's development, including academics, behavior, social and emotional needs, and attendance. At its heart, MTSS is about delivering high-quality mathematics instruction, monitoring progress, and engaging all students in meaningful, grade-level learning.

An effective MTSS framework for numeracy includes:

- Tier 1 (Core Instruction): High-quality, evidence-based instruction delivered to all students. This tier is culturally and linguistically responsive, differentiated to meet diverse needs, and aligned across grade levels. Tier 1 instruction emphasizes reasoning, problem solving, and application, which are essential components of numeracy.
- Tier 2 (Targeted Supports): Small-group or supplemental instruction is provided for students who need additional support as well as opportunities for extension and enrichment. These supports are guided by assessment data and include reteaching of concepts, structured practice, and scaffolded tasks that reinforce mathematical reasoning and confidence while addressing prerequisite skill gaps necessary for grade-level mastery. In addition, students who demonstrate readiness are provided with enrichment experiences such as problem-based learning, mathematical investigations, and opportunities to apply concepts in more complex or accelerated contexts to deepen understanding. All supports and extensions are aligned to Tier 1 core instruction to ensure coherence and continuity of learning.
- Tier 3 (Intensive Interventions or Acceleration): Individualized, intensive supports are provided for students with significant, persistent prerequisite skill gaps. These interventions are evidence-based, tailored to student strengths, and aligned to Tier 1 core instruction, with a focus on closing foundational gaps and building readiness for grade-level learning. Extension and enrichment opportunities are provided for students who demonstrate advanced readiness or are identified as gifted. These opportunities include complex problem-solving, higher-level tasks, and experiences that deepen and extend learning through advanced or accelerated applications, while remaining aligned to Tier 1 core instruction.
- Engagement and Belonging: MTSS also addresses student engagement, persistence, and sense of belonging in mathematics classrooms. When students feel safe, valued, and included, they are more likely to participate fully in learning opportunities and develop positive mathematical identities.
- Collaboration: Effective MTSS requires collaboration across general educators, interventionists, special educators, administrators, and families. Working together ensures that supports are coherent, timely, and aligned to each student's developmental needs.

By emphasizing both a strong core and tiered supports, MTSS helps prevent gaps in numeracy development while ensuring that interventions are available and equitable. It ensures that every student, including those with specialized needs, MLs, and those historically marginalized in mathematics, has access to high-quality instruction and the opportunity to thrive.<sup>47</sup>



The multi-level prevention system provides increasingly intense levels of instruction and support to address student need.

# What is Multi-Level Prevention System?

- More intensive than Tier 2
- Individualized to address student need based on student data
- Aligned with Tier 1 on a case-by-case basis
- Optimal group size and dosage based on student need
- Led by well-trained staff

- Standardized and evidence-based intervention provided with fidelity that are matched to student needs
- Complements and in addition to Tier 1
- Led by staff trained on the intervention
- Optimal group size and dosage

- Articulated learning objectives among classrooms and from one grade-level to the next
- Research based curriculum aligned with state standards
- Delivered using evidence-based instructional strategies
- Adequate time to teach content and for practice with skills
- Data-driven differentiated instruction

## Tier 1

**Universal supports provided to all students.**

## Tier 2

**Targeted supports provided to 10–15% of students.**

## Tier 3

**Intensive supports provided to 3–5% of students.**

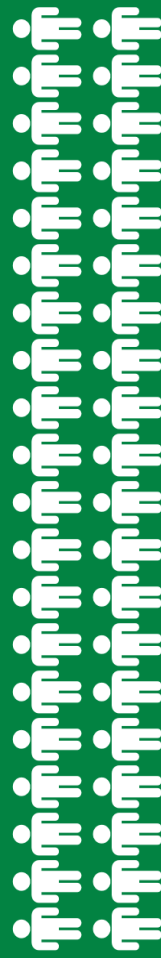
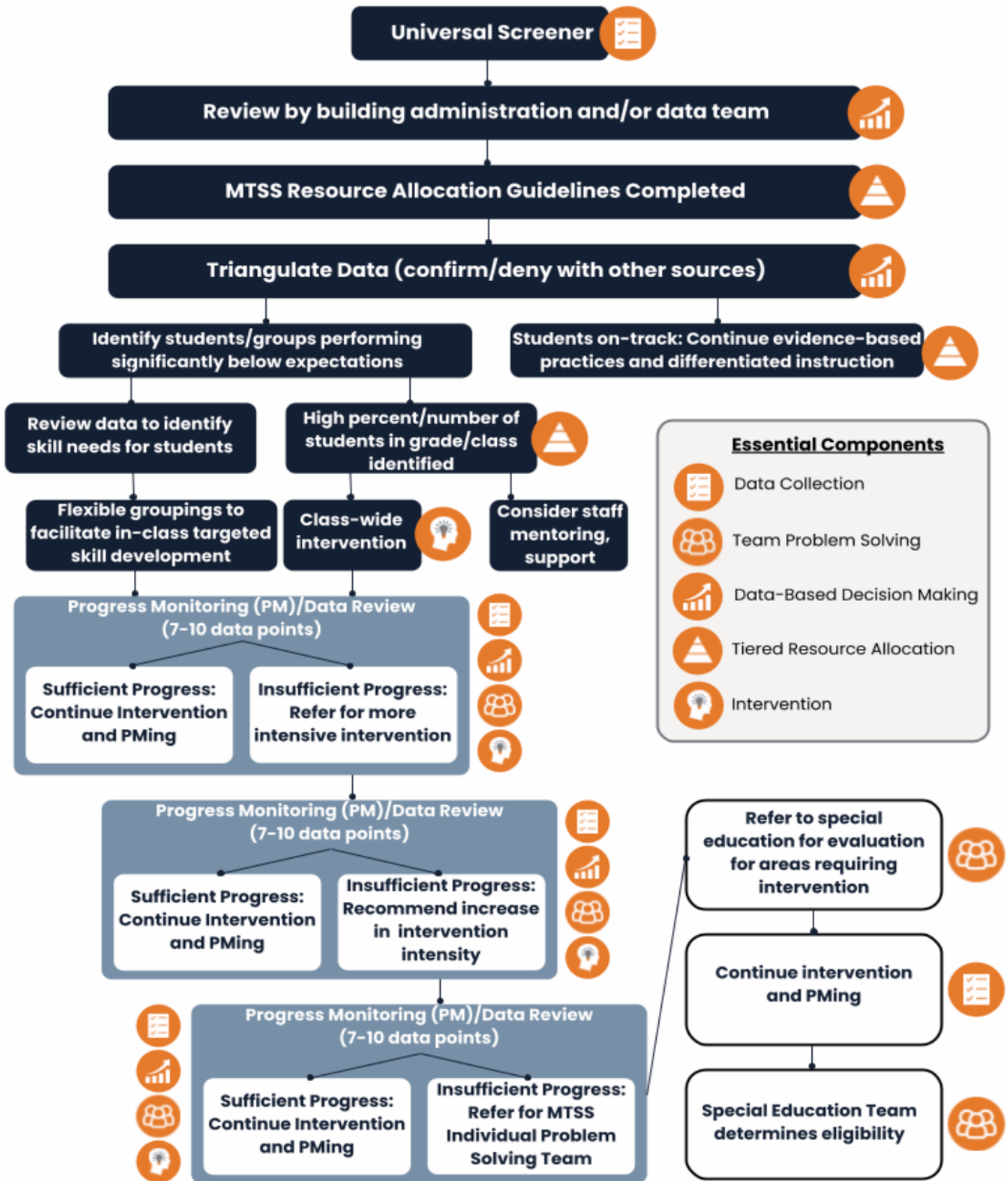


Figure 13: MTSS Tiers

From National Center on Intensive Intervention at American Institutes for Research. [Essential Components of MTSS: Infographic Collection](#). Washington, DC: NCI.



\*Not all areas of special education eligibility require the RtI process. This represents an ideal intervention framework; there may be individual exceptions requiring a referral for special education prior to when this model prescribes.

Figure 14: MTSS Intervention Framework

From Illinois Specific Learning Disability Support Project. [The Dyscalculia Handbook](#), 2026.

## Assessments to Support Numeracy

Assessment is the primary tool for informing instructional decision-making in mathematics, including planning, differentiation, intervention, placement, grading, and progression. It shapes decisions about student learning needs, curriculum implementation, and, in some instances, funding allocation.

ISBE's [performance level descriptors](#) (PLDs) as well as samples to success can assist educators in supporting student learning and assessment. PLDs are designed to bridge the state assessment to classroom instruction and the systems of formative assessments to inform instruction as well as individual student need.

## Universal Design for Learning

Universal Design for Learning (UDL) is a framework that promotes equitable, rigorous learning for all students. According to the Center for Applied Special Technology (CAST), "UDL aims to change the design of the environment rather than to situate the problem as a perceived deficit within the learner."<sup>48</sup>

The UDL framework consists of three guidelines:

**Engagement**  
*(the why of learning)*

**Representation**  
*(the what of learning)*

**Action and expression**  
*(the how of learning)*

By embedding UDL into assessment practices, educators ensure that every learner has equitable opportunities to demonstrate what they know and can do. Traditional assessments often assume that all students can access content, instructions, and response formats in the same way, but this approach unintentionally creates barriers. UDL challenges that assumption by proactively designing assessments that provide multiple pathways for access and multiple ways to express understanding. When assessments are designed through the UDL lens, students encounter content that is accessible from the start. This might mean offering questions with visual supports, read-aloud options, translations, or flexible text formats so that the focus remains on the construct being measured rather than on a student's ability to navigate inaccessible text. Similarly, students are given opportunities to respond in ways that match their strengths. For some, this may include using speech-to-text tools, drawing or modeling solutions, or creating digital artifacts to represent their thinking. These flexible approaches do not dilute rigor; instead, they ensure that the assessment is measuring the intended skill or concept rather than a barrier unrelated to the learning target.<sup>49</sup>



## Application of UDL in the Mathematics Classroom<sup>50</sup>:

- Use formative assessment data regularly (e.g., exit tickets, observations, student work) to make immediate instructional adjustments such as regrouping, reteaching, or extending tasks based on evidence of understanding
- Design engaging, relevant assessment tasks with multiple entry points and allow students to show understanding using varied representations (e.g., models, equations, diagrams, explanations, or manipulatives)
- Look at student thinking, not just answers, to identify partial understanding and guide next steps in instruction
- Provide feedback that focuses on mathematical reasoning and strategies, not only correctness
- Plan instruction based on the idea that students will approach mathematics in different ways, rather than expecting a single method for all learners
- Provide structured opportunities for students to collaborate with peers during problem solving, allowing them to share strategies and build understanding through mathematical discourse
- Intentionally create a supportive learning environment that acknowledges and builds on student strengths, normalizes mistakes, and provides low-risk opportunities for participation to help reduce math anxiety and increase confidence in engaging with challenging tasks





# CAST Universal Design for Learning Guidelines

The goal of UDL is learner agency that is purposeful & reflective, resourceful & authentic, strategic & action-oriented.

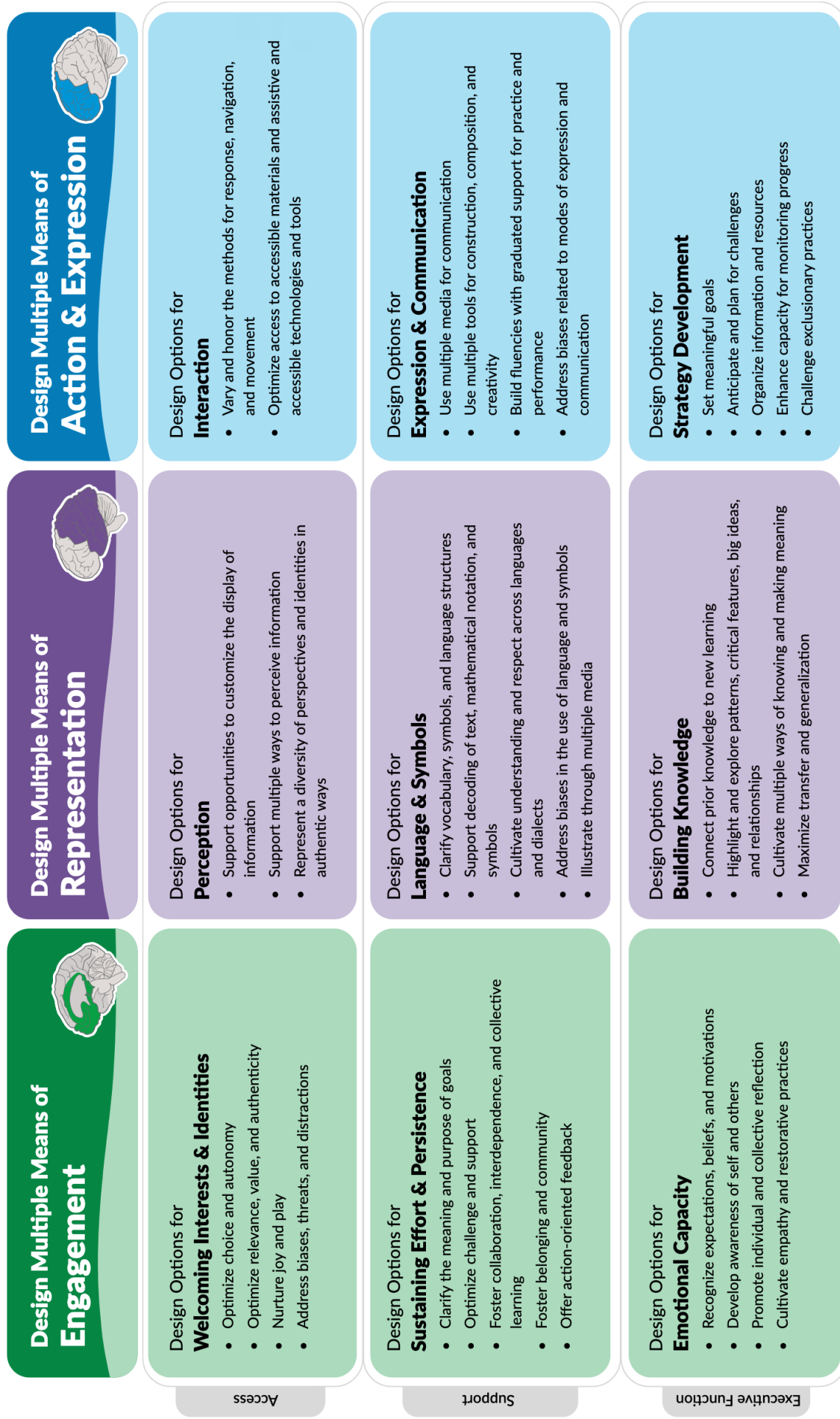


Figure 15: Universal Design for Learning Guidelines

From CAST. [Universal Design for Learning Guidelines \(Version 3.0\): Graphic Organizer](#). Wakefield, MA: CAST, n.d.

## Assessment Types

To effectively support teaching and learning, educators should use a combination of equitable, high-quality, and standards-aligned assessment types. It is important to recognize that not all assessments are created equal; educators must consider the purpose of assessments as well as the appropriate time to administer them.

Type	Purpose	Key Components	Best Practices
<b>Universal Screening</b>	Universal screening assists in identifying students who are thriving, at risk, or in need of acceleration through a systematic evaluation of all students within a class, grade, school building, or school district that focuses on critical academic and social-emotional indicators.	<ul style="list-style-type: none"> <li>Administered to all students at the beginning of the year</li> <li>Concise assessment</li> </ul>	<ul style="list-style-type: none"> <li>Use reliable, valid tools</li> <li>Establish goals and methods of screening prior to administering</li> <li>Develop a protocol for how data will be used to assess curriculum, instruction, educational environments, etc.</li> </ul>
<b>Benchmarking</b>	Benchmark assessment is a cyclical process that involves using a screening tool multiple times throughout the school year to monitor students' responses to core instruction.	<ul style="list-style-type: none"> <li>Administered three times throughout the school year</li> <li>Defines expected skill levels for students at each grade level at specific times throughout the year</li> </ul>	<ul style="list-style-type: none"> <li>Use data to analyze common learning trends</li> <li>Use results to inform interventions</li> <li>Include students in goal setting and communicate benchmark results with parents in a digestible manner</li> </ul>
<b>Diagnostic Screening</b>	Diagnostic screeners pinpoint specific pre-grade and grade-level skill gaps to guide appropriate Tier 2 supports and Tier 3 intensive interventions that address unfinished learning while aligning with Tier 1 instruction. These screeners also identify students who are ready for accelerated learning.	<ul style="list-style-type: none"> <li>Must identify and measure student needs and strengths</li> </ul>	<ul style="list-style-type: none"> <li>Use results to inform interventions and acceleration</li> <li>Use results to inform specific pre-grade and grade-level skills deficits</li> </ul>

Type	Purpose	Key Components	Best Practices
<b>Progress Monitoring</b>	Progress monitoring measures the impact of assigned interventions by tracking progress toward grade-level mastery throughout the intervention period.	<ul style="list-style-type: none"> <li>Regularly monitors progress toward mastery of pre-grade and grade-level skill gaps identified through a student's diagnostic screener</li> <li>Administered at least every two weeks</li> </ul>	<ul style="list-style-type: none"> <li>Use results to immediately guide coherent, individualized instructional decisions in alignment with each tier of MTSS</li> </ul>
<b>Formative Assessments</b>	Formative assessments are used during the learning process to gather evidence about student understanding and to guide immediate instructional adjustments.	<ul style="list-style-type: none"> <li>Can take place multiple times throughout a lesson in forms such as observation, exit tickets, quizzes, discourse, etc.</li> </ul>	<ul style="list-style-type: none"> <li>Provide timely feedback and use results to guide instruction throughout the remainder of the learning unit</li> <li>Promote student reflection</li> </ul>
<b>Summative Assessments</b>	Summative assessments evaluate student learning at the conclusion of an instructional period, often for comparison or accountability purposes.	<ul style="list-style-type: none"> <li>Standards-aligned</li> <li>Can include a unit test, projects, performance tasks, student portfolios, etc.</li> </ul>	<ul style="list-style-type: none"> <li>Students are aware of assessment criteria</li> <li>Data informs future instruction</li> </ul>
<b>State Assessment</b>	State assessments measure the learning of state standards and are often used as accountability measures.	<ul style="list-style-type: none"> <li>Standardized, large-scale assessment</li> </ul>	<ul style="list-style-type: none"> <li>Provide students with appropriate test-taking strategies</li> <li>Focus on standards-aligned instruction throughout entirety of school year</li> </ul>

Figure 16 Assessment Types

Assessment is only meaningful when it informs and changes instructional decision-making. The purpose of assessment is not simply to collect data but to guide what happens next in teaching and learning. Evidence of student thinking should lead to intentional instructional adjustments, such as modifying pacing, revisiting concepts, grouping students strategically, or selecting different tasks and representations. There is a critical distinction between assessment as a tool for decision-making and assessment as mere data collection; without a clear instructional response, assessment does not improve student outcomes. When used effectively, assessment becomes an ongoing process that drives responsive instruction and ensures all students have access to meaningful mathematical learning.

## **Section Summary**

The Framework for the Evidence-Based Development of Numeracy skills establishes the instructional foundation of the Illinois Comprehensive Numeracy Plan. It emphasizes that developing numeracy requires intentional, evidence-based instruction that connects content, practices, and student thinking across a cohesive continuum of learning. By integrating the Six Components of Numeracy with high-quality instructional practices, educators create opportunities for students to build understanding, develop fluency, and communicate and apply their knowledge in meaningful ways. Attention to instructional considerations ensures that all learners have access to rigorous mathematics through inclusive approaches, while assessment practices provide ongoing evidence to guide and refine instruction. Together, these elements illustrate what high-quality, evidence-based numeracy instruction entails and how it can be developed and sustained across Illinois classrooms.

# Additional Resources

## Section 1

These resources are recommended based on feedback and insights gathered through public engagement that demonstrate strong alignment to this section of the Numeracy Plan. They provide opportunities to explore the topics in greater depth and offer additional context to support the work. Their inclusion does not constitute an endorsement by ISBE.

### Books

- *5 Practices for Orchestrating Productive Mathematics Discussions* by Margaret Schwan Smith and Mary Kay Stein
- *Adding it Up: Helping Children Learn Mathematics* by the National Research Council
- [The Dyscalculia Handbook](#) by the Illinois Specific Learning Disability Support Project
- *Figuring Out Fluency* series by Jennifer M. Bay-Williams and John J. SanGiovanni
- *The Impact of Identity in K–8 Mathematics: Rethinking Equity-Based Practices* by Julia Aguirre, Karen Mayfield-Ingram, and Danny Martin
- *Making Number Talks Matter: Developing Mathematical Practices and Deepening Understanding* by Cathy Humphreys and Ruth Parker
- *Math Fact Fluency: 60+ Games and Assessment Tools to Support Learning and Retention* by Jennifer Bay-Williams and Gina Kling
- *More Than Counting: Math Activities for Preschool and Kindergarten* by Sally Moomaw and Brenda Hieronymus
- *Number Sense Routines* series by Jessica F. Shumway
- *Number Talks* series by Sherry Parrish
- *Principles to Actions: Ensuring Mathematical Success for All* by DeAnn Huinker, Melissa H. Clements, Douglas Brahier, and Steve Leinwand
- *Rethinking Disability and Mathematics: A UDL Math Classroom Guide for Grades K–8* by Rachel Lambert
- *Rough Draft Math: Revising to Learn* by Amanda Jansen
- *Where's the Math? Books, Games, and Routines to Spark Children's Thinking* by Laura Grandau and Mary Hynes-Berry

# Additional Resources

## Section 1

### Podcasts

- *Adding It All Up* from NCTM
- *Making Math Moments That Matter* hosted by Kyle Pearce and Jon Orr
- *Math Ed Podcast* hosted by Samuel Otten
- *Math Teacher Lounge* hosted by Bethany Lockhart Johnson and Dan Meyer
- *There's Power in Teaching* from PDK International
- *This Teacher Life* hosted by Monica Genta

### Tools

- [Inside Mathematics](#)
- [Learning Trajectories](#)
- [Math concept progression videos](#) by Graham Fletcher
- [Mathematics instructional routines](#)
- [Mathematics vocabulary cards](#)
- [Number strings resources](#)
- [Problem Strings](#)



# **Goal 1: Workbook**

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**Students will build and apply numeracy skills through tasks that develop reasoning, fluency, and real-world problem solving.**

## Goal 1

Students will build and apply numeracy skills through tasks that develop reasoning, fluency, and real-world problem solving.

### Implementation Considerations

- Classroom environments should promote mathematical reasoning, discourse, and flexible problem solving.
- Students should have access to high-quality instructional materials that build conceptual understanding and procedural fluency.
- Instruction should incorporate multiple representations, including concrete, representational, and abstract models.
- Assessment practices should guide instruction and help identify gaps in conceptual understanding.
- Teachers need ongoing support to implement evidence-based mathematics practices with consistency across grade levels.

## Notes

### Next Steps

Use high-quality instructional materials and tasks that promote reasoning and conceptual understanding.

Incorporate mathematical discourse routines to help students communicate thinking clearly.

Apply the CRA progression to support students in building strong numeracy foundations.

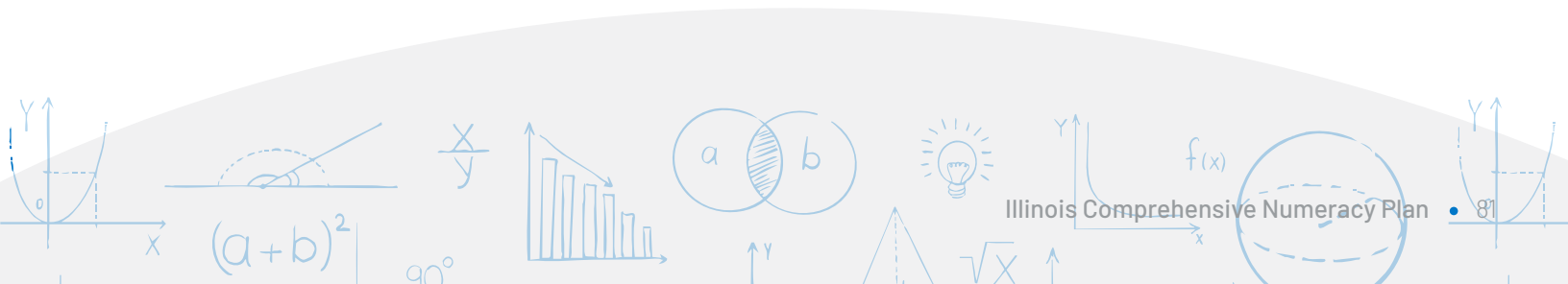
Collaborate with your grade-level team to strengthen coherence across units and topics.

Monitor student progress using formative assessments and adjust instruction to address misconceptions.



## Reflection Questions

1. What does a numeracy-rich classroom environment look and sound like in my grade level?
2. How do I ensure all students access grade-level mathematics content with appropriate scaffolds?
3. What strategies do I use to identify and address common mathematical misconceptions?
4. How do I ensure my assessment practices capture both conceptual understanding and procedural skills?
5. What obstacles impact my ability to implement evidence-based numeracy instruction with fidelity?



## Goal 1

Students will build and apply numeracy skills through tasks that develop reasoning, fluency, and real-world problem solving.

### Implementation Considerations

- Classroom leaders should help educators strengthen instruction using evidence-based numeracy practices.
- Collaboration across grade levels supports coherent progressions and reduces instructional fragmentation.
- Strong data-analysis routines help identify instructional needs and trends across classrooms.
- Curriculum and resources must align with the components of numeracy and grade-level standards.
- Teacher leaders should model and reinforce the use of multiple representations and mathematical discourse.

### Notes

### Next Steps

Support educators in implementing high-quality tasks that promote reasoning and problem solving.

Facilitate vertical team discussions to strengthen alignment across grade bands.

Help teachers analyze student work to identify strengths, misconceptions, and instructional needs.

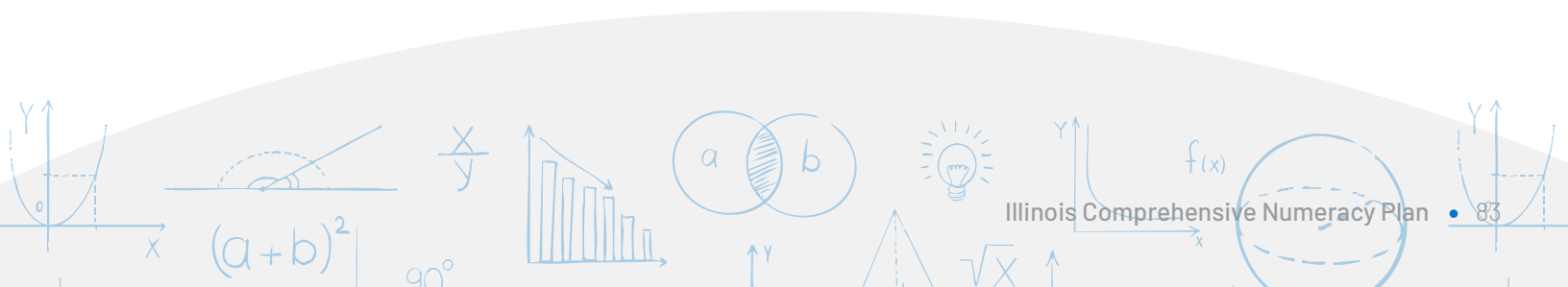
Communicate instructional successes and needs with administrators to inform professional learning.

Guide teachers in using data to adjust instruction and monitor student numeracy development.



## Reflection Questions

1. How are teachers supported in using evidence-based mathematics instruction practices?
2. How do I help maintain alignment and coherence across grade-level teams?
3. What trends do I observe in student understanding across classrooms?
4. How do I best support teachers in identifying and addressing misconceptions?
5. What barriers exist in helping teachers implement high-quality numeracy instruction?



## Goal 1

Students will build and apply numeracy skills through tasks that develop reasoning, fluency, and real-world problem solving.

### Implementation Considerations

- Effective numeracy instruction requires scheduling structures, materials, and training that support strong implementation.
- Equitable access to high-quality mathematics curriculum is essential for student success.
- School-level assessment systems should provide timely data to guide instruction and interventions.
- Collaborative planning time strengthens coherence and instructional quality.
- School leaders need to foster a culture where all educators and students see themselves as capable mathematical thinkers.

## Notes

### Next Steps

Engage teacher teams in reviewing student data to understand numeracy strengths and gaps.

Evaluate current mathematics curriculum and intervention materials and identify areas for improvement.

Strengthen MTSS processes to identify students needing additional numeracy support.

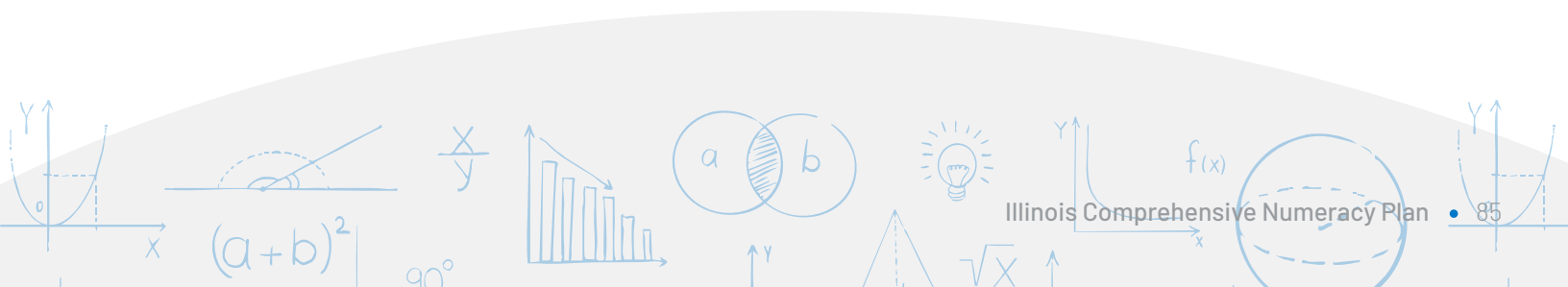
Build assessment literacy to ensure staff administer and interpret data effectively.

Provide ongoing professional learning aligned to evidence-based mathematics instruction.



## Reflection Questions

1. Are all students receiving high-quality, evidence-based numeracy instruction?
2. Does the school's curriculum align to the components of numeracy described in the Illinois Comprehensive Numeracy Plan?
3. How effectively does the school use data to guide instruction and interventions?
4. What systems support teacher collaboration and coherence in mathematics instruction?
5. How do school structures promote equitable mathematics learning opportunities?



## Goal 1

Students will build and apply numeracy skills through tasks that develop reasoning, fluency, and real-world problem solving.

### Implementation Considerations

- District numeracy plans should reflect student data and instructional needs across schools.
- Curriculum adoption processes should prioritize high-quality, standards-aligned mathematics materials.
- Assessment systems must support monitoring of conceptual understanding, procedural skills, and problem solving.
- Vertical coherence is critical from early childhood through high school mathematics pathways.
- Professional learning should be aligned to evidence-based mathematics instruction and accessible across the district.

## Notes

### Next Steps

Prioritize selection and implementation of high-quality mathematics instructional materials.

Analyze districtwide data to assess achievement and identify areas requiring targeted support.

Strengthen assessment plans to ensure appropriate measures are available across grade levels.

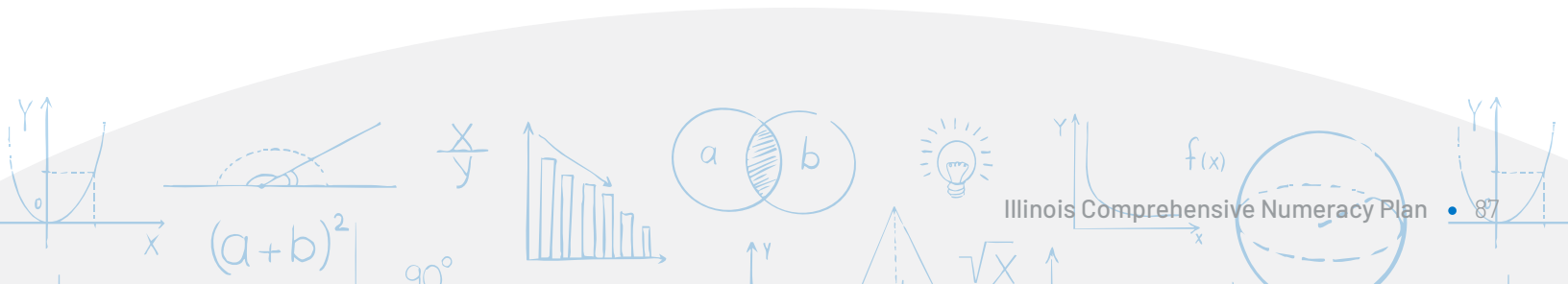
Guide schools in building strong MTSS structures for numeracy.

Identify and reduce inequities in access to resources, coursework, and instructional opportunities.



## Reflection Questions

1. What does district data reveal about numeracy outcomes and equity across schools?
2. How well does the district's curriculum support evidence-based numeracy instruction?
3. What instructional and assessment practices are currently successful?
4. Which grade levels or student populations require additional support?
5. How does the district ensure coherence from early numeracy through advanced coursework?



## Goal 1

Students will build and apply numeracy skills through tasks that develop reasoning, fluency, and real-world problem solving.

### Implementation Considerations

- Regional support should reflect the varied needs of districts and communities. Regional leaders play a critical role in providing professional learning aligned with evidence-based numeracy practices.
- Curriculum review cycles can be strengthened through regional collaboration.
- Data across districts can illuminate regional patterns and resource needs.
- Regional efforts should prioritize equitable access to high-quality mathematics instruction.

## Notes

### Next Steps

Conduct regional needs assessments to guide professional learning offerings.

Support districts in analyzing numeracy data and addressing instructional gaps.

Facilitate opportunities for districts to review mathematics curriculum and share best practices.

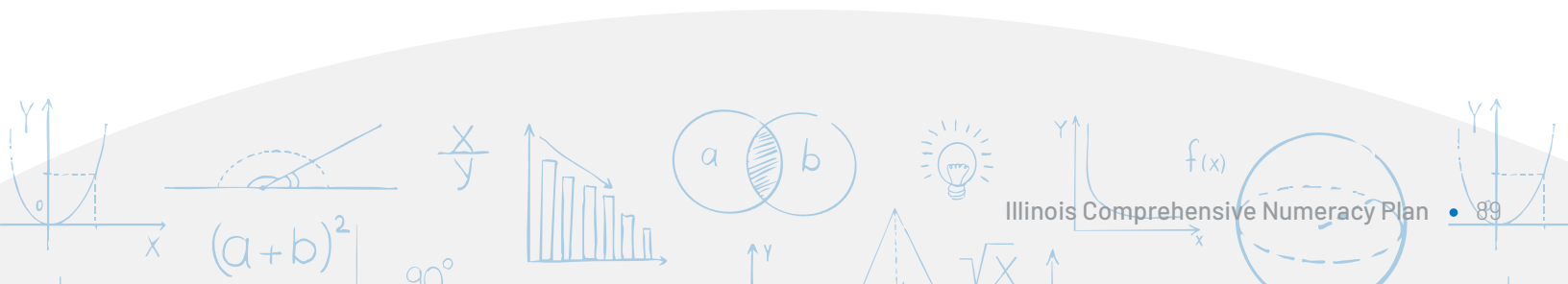
Provide guidance on selecting and implementing evidence-based instructional materials.

Offer targeted support for districts serving high-poverty or high-need student populations.



## Reflection Questions

1. How will regional leaders identify and respond to district-level numeracy needs?
2. What structures can strengthen regional collaboration around mathematics instruction?
3. What professional learning supports are most needed across the region?
4. How can regional leaders ensure equitable access to high-quality resources?
5. What barriers limit regional support for numeracy improvement?



## Goal 1

Students will build and apply numeracy skills through tasks that develop reasoning, fluency, and real-world problem solving.

### Implementation Considerations

- Preparation programs play a foundational role in developing teachers' mathematical content and pedagogical knowledge.
- Candidates must understand numeracy as outlined in the Illinois Comprehensive Numeracy Plan, including reasoning, fluency, and real-world application.
- Coursework should model evidence-based mathematics instruction and UDL principles.
- Early field experiences should reflect high-quality, coherent mathematics practices.
- Preparation programs help shape candidates' beliefs about mathematical identity and capability.

## Notes

### Next Steps

Align coursework with the Illinois Comprehensive Numeracy Plan's numeracy definition and components.

Provide clinical experiences that reflect evidence-based mathematics instruction.

Strengthen candidate learning around diagnosing and addressing misconceptions.

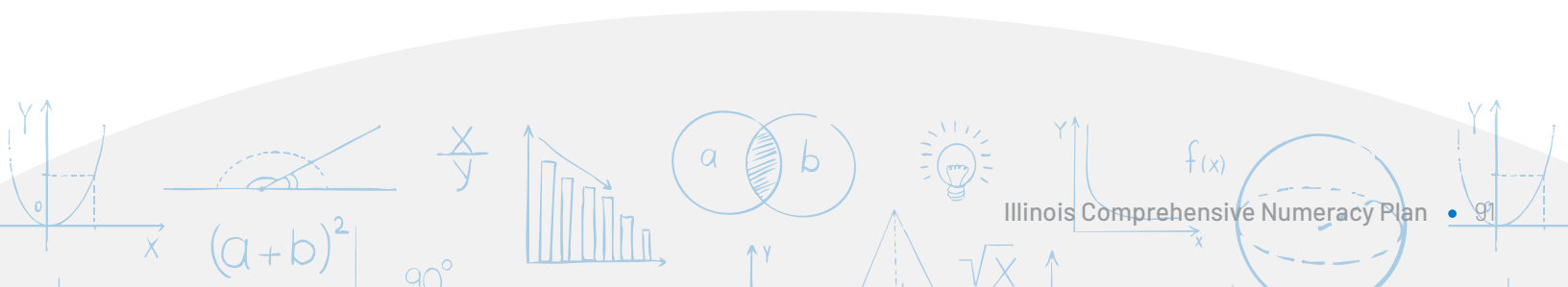
Incorporate instruction on mathematical discourse, multiple representations, and problem-solving frameworks.

Build partnerships with districts to align expectations and practice.



## Reflection Questions

1. How do educator preparation programs introduce candidates to numeracy development?
2. What opportunities do candidates have to observe and practice evidence-based mathematics instruction?
3. How are field placements aligned to high-quality mathematics teaching?
4. How do programs prepare candidates to support diverse learners in mathematics?
5. What additional supports are needed to strengthen candidates' mathematical knowledge for teaching?



## Goal 1

Students will build and apply numeracy skills through tasks that develop reasoning, fluency, and real-world problem solving.

### Implementation Considerations

- State leaders must ensure equitable access to high-quality mathematics instruction across Illinois.
- Statewide data should guide resource allocation and policy decisions.
- High-quality instructional materials and professional learning should be prioritized.
- State initiatives should promote coherence across districts and regions.
- Policies must reflect the Illinois Comprehensive Numeracy Plan's commitment to equity and evidence-based mathematics instruction.

## Notes

### Next Steps

Communicate the numeracy plan and support stakeholders with aligned resources.

Provide tools for evaluating high-quality mathematics curriculum and interventions.

Monitor statewide numeracy data to identify trends and inequities.

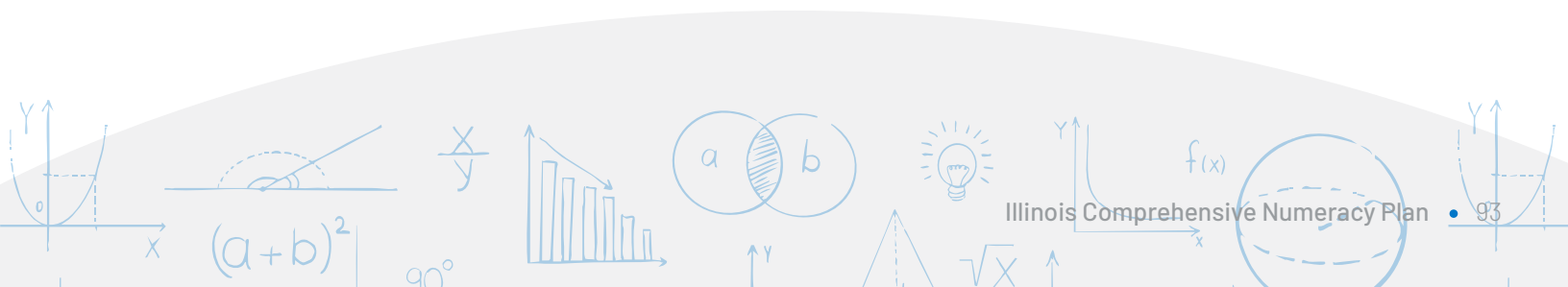
Allocate resources to support schools and districts with the greatest need.

Ensure professional learning reflects the most current evidence-based mathematics practices.



## Reflection Questions

1. How will state leaders promote equitable access to high-quality mathematics instruction?
2. What statewide data trends require immediate attention?
3. How can state policies remove barriers to effective numeracy instruction?
4. How can state leaders support districts in selecting high-quality mathematics materials?
5. What obstacles may limit statewide implementation of the Illinois Comprehensive Numeracy Plan?



## Goal 1

Students will build and apply numeracy skills through tasks that develop reasoning, fluency, and real-world problem solving.

### Implementation Considerations

- Families and communities contribute to building positive mathematical identities.
- Access to community-based numeracy resources can strengthen learning beyond school.
- Transparent communication about student progress fosters partnership.
- Community organizations can support real-world mathematics experiences.
- Equitable access to numeracy opportunities varies across communities and should be addressed.

## Notes

### Next Steps

Ask questions and engage with schools to understand numeracy expectations.

Encourage mathematical thinking through everyday activities at home and in the community.

Advocate for accessible resources that support numeracy learning for all students.

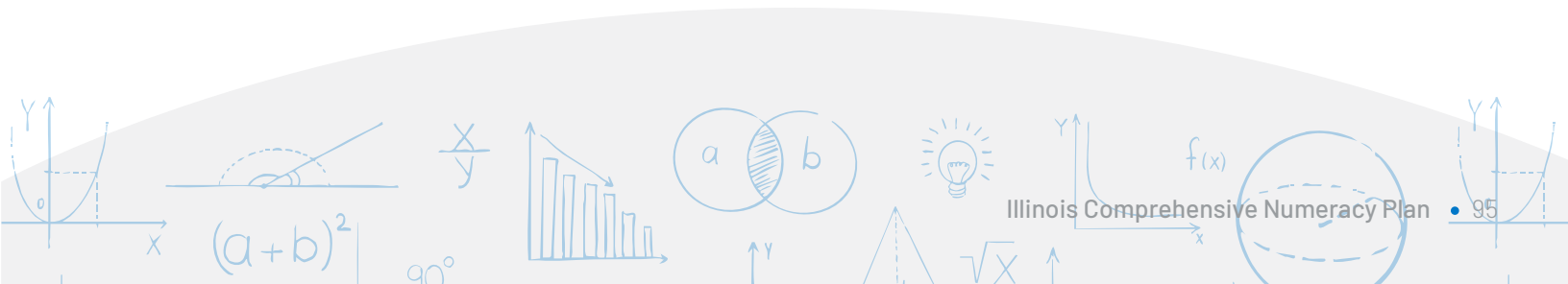
Participate in or support programs that strengthen family-school partnerships.

Review assessment information to better understand student strengths and needs.



## Reflection Questions

1. How can families and community partners support numeracy development at home?
2. What resources would help families better understand mathematics instruction?
3. How can communities create meaningful real-world mathematics experiences for students?
4. How can partnerships between families and schools be strengthened?
5. What barriers limit community engagement in supporting numeracy?



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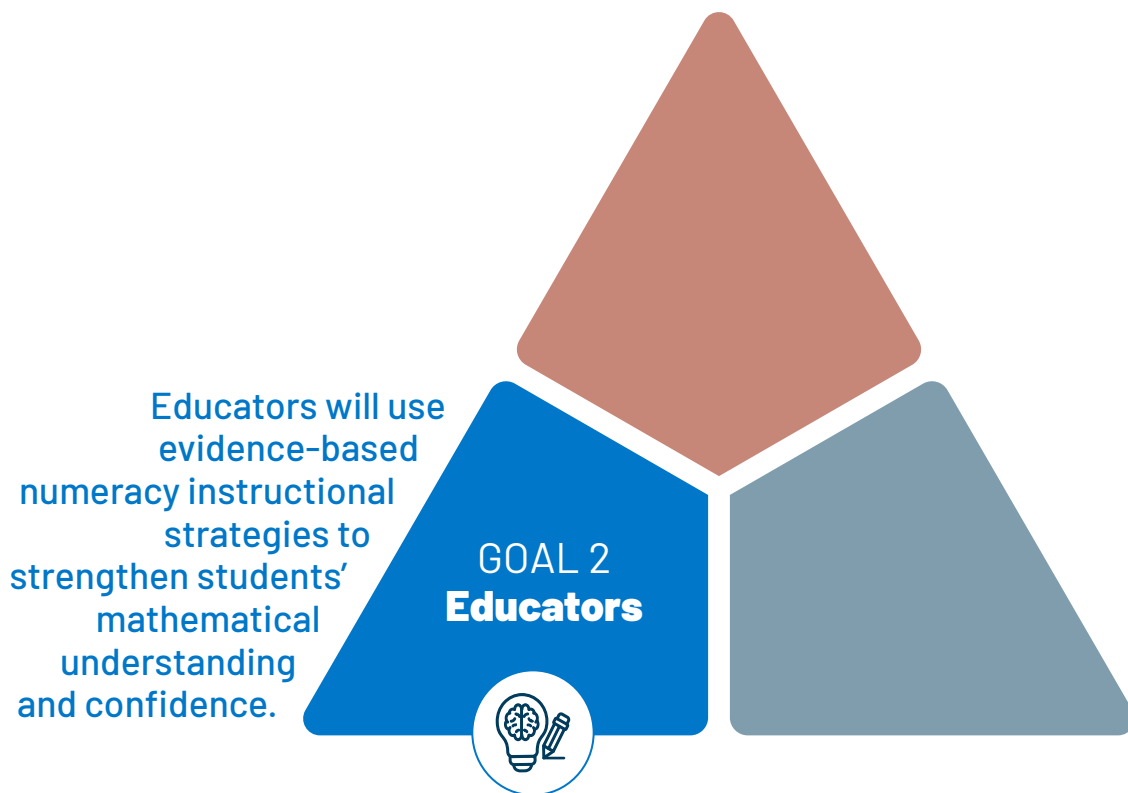
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# EDUCATOR PROFESSIONAL LEARNING AND DEVELOPMENT



This section is dedicated to **Goal 2**:



This section of the plan will cover the following main topics:

1. The Professional Learning Continuum

2. Equity in Professional Learning

3. The Focus of Professional Learning

4. School and District Leaders

5. In-Service Educators

6. Teacher Preparation

## Section Overview

This section focuses on the systems and structures necessary to develop and sustain high-quality mathematics instruction through professional learning. It begins by defining the essential characteristics of effective professional learning and emphasizes the importance of aligning learning opportunities to evidence-based instructional practices. The section then outlines five interrelated domains that guide educator growth, ensuring alignment with the Six Components of Numeracy and the instructional priorities described in Section 1. Central to this work is the professional learning continuum, which recognizes that educator development spans from preparation programs through ongoing in-service learning and leadership roles. Finally, this section highlights the role of administrators in creating coherent, responsive systems of support.

## The Professional Learning Continuum

Professional learning in mathematics plays a central role in achieving Illinois' vision in supporting the development of mathematically literate citizens prepared for the demands of the 21st century. Students thrive when educators engage in sustained, evidence-based learning that deepens mathematical knowledge, strengthens teaching practices, and affirms professional and mathematical identities.<sup>1</sup> Professional learning clarifies a shared, evidence-based definition of numeracy: the ability for all students to confidently understand, interpret, and apply mathematical concepts across all domains of mathematics in a variety of real-world and academic contexts. With that lens, teachers design instruction that advances reasoning, problem solving, and confidence.<sup>2</sup>

This section addresses educator learning across the continuum, from school and district leaders to educator preparation programs (EPPs). High-quality professional learning does not occur in isolated stages. Instead, it forms a continuum that begins in educator preparation programs, deepens through in-service professional learning, and is sustained and extended by school and district leadership. Each stage builds on the last and depends on the others to achieve Illinois' vision for ambitious and equitable numeracy instruction.

### Leadership

#### SUPPORT

By engaging in professional learning themselves, providing non-evaluative feedback, and sustaining collaborative structures like PLCs and data teams, leaders ensure that high-quality professional learning becomes part of the fabric of every school and district.<sup>3</sup>

### In-Service Educators

#### REFINEMENT

Through job-embedded professional learning like coaching, Professional Learning Communities (PLCs), mentoring, institutes, and equity-focused professional learning, teachers refine their practice, respond to student needs, and grow as instructional leaders. Sustained engagement ensures that teachers continue to develop across their careers.<sup>4</sup>

### Educator Preparation Programs

#### FOUNDATION

Candidates build mathematical knowledge, develop inclusive practices, and cultivate positive identities as mathematical thinkers. They learn the habits of mind and pedagogical strategies that allow them to launch their careers with confidence.

Category	Leadership	In-Service Educators	Educator Preparation Programs (EPPs)
<p><b>Content Knowledge</b></p>	<ul style="list-style-type: none"> <li>• Provide and actively participate in professional learning through institutes, Professional Learning Communities (PLCs), coaching cycles, and mentoring alongside educators to model shared commitment and extend knowledge of numeracy</li> <li>• Strengthen teacher content knowledge in applied contexts, including interdisciplinary connections to science, career and technical education, and social sciences</li> </ul>	<ul style="list-style-type: none"> <li>• Actively participate in professional learning through institutes, PLCs focused on numeracy, coaching cycles, and mentoring</li> <li>• Align numeracy instruction and assessments to Illinois Learning Standards for mathematics and SMPs</li> </ul>	<ul style="list-style-type: none"> <li>• Align coursework and clinical experiences with Illinois endorsement standards (23 Ill. Adm. Code 20, 21, 26, 27) and Illinois Licensure Testing System (ILTS) frameworks</li> <li>• Ensure pre-service teachers develop strong content mastery alongside a foundational understanding of mathematical knowledge for teaching</li> </ul>
<p><b>Instructional Best Practices and Pedagogy</b></p>	<ul style="list-style-type: none"> <li>• Refine and extend these practices through lesson study, coaching cycles, and induction programs</li> <li>• Support implementation through feedback tools</li> <li>• Align leadership with <a href="#">Professional Standards for Educational Leaders</a> and <a href="#">Illinois Performance Standards for School Leaders</a></li> </ul>	<ul style="list-style-type: none"> <li>• Engage in instructional routines (number talks, problem strings, etc.) as a learner</li> <li>• Analyze student work collaboratively to identify reasoning and misconceptions</li> <li>• Seek feedback from instructional coaches or content area teams</li> </ul>	<ul style="list-style-type: none"> <li>• Introduce evidence-based practices as instructional approaches supported by rigorous, peer-reviewed research demonstrating a positive impact on student learning</li> <li>• Provide examples such as those outlined by <a href="#">IES</a></li> </ul>



Category	Leadership	In-Service Educators	Educator Preparation Programs (EPPs)
<b>Equity</b>	<ul style="list-style-type: none"> <li>• Embed Culturally Responsive Teaching and Learning Standards (CRTL) and UDL strategies into ongoing professional learning</li> <li>• Prioritize equity in resource allocation and ensure marginalized students receive targeted support</li> </ul>	<ul style="list-style-type: none"> <li>• Apply CRTL and UDL principles in lesson design</li> <li>• Differentiate instruction for diverse learners</li> <li>• Advocate for equitable access to resources</li> </ul>	<ul style="list-style-type: none"> <li>• Cultivate dispositions that affirm equity and inclusivity</li> </ul>
<b>Mentoring</b>	<ul style="list-style-type: none"> <li>• Offer induction programs and mentoring that reinforce effective practices</li> <li>• Guarantee teachers have protected time and resources for PLCs and data analysis</li> </ul>	<ul style="list-style-type: none"> <li>• Participate in mentoring and peer observation</li> <li>• Use data teams to analyze student progress</li> <li>• Reflect on instructional practices</li> </ul>	<ul style="list-style-type: none"> <li>• Provide clinical practice where candidates rehearse teaching, receive feedback, and analyze student learning</li> </ul>
<b>Continuous Improvement</b>	<ul style="list-style-type: none"> <li>• Evaluate professional learning by collecting feedback, monitoring classroom practice, and analyzing outcomes</li> <li>• Use data from observations and PLCs to guide improvement</li> </ul>	<ul style="list-style-type: none"> <li>• Document growth through portfolios</li> <li>• Share evidence of student learning during PLCs</li> <li>• Engage in continuous improvement cycles</li> </ul>	<ul style="list-style-type: none"> <li>• Require integrative experiences, such as capstones, seminars, or portfolios that demonstrate readiness by connecting content, pedagogy, and equity</li> <li>• Provide opportunities for candidates to self-evaluate their teaching practices and set goals for improvement</li> </ul>

Figure 17: Continuum of Professional Learning

## Equity in Professional Learning

Equity serves as the foundation of professional learning in mathematics and must be intentionally imbedded across the continuum of educator professional learning.<sup>5</sup> Professional learning should integrate culturally responsive pedagogy, UDL strategies, and tools for supporting multilingual learners and students with disabilities.<sup>6</sup> Administrators play a critical role in ensuring equitable access to high-quality professional learning through intentional allocation of time, resources, and supports. Equity-focused leadership includes using data-based decision making, involving educators in the planning of professional learning, and monitoring the implementation of programs and initiatives to ensure professional learning benefits all students. In-service educators must be provided with equitable professional learning that focuses on differentiation, student data, and asset-based approaches.<sup>7</sup> Educators can also attend to equity by engaging in collaborative learning and identifying and addressing instructional barriers for underserved students. At the pre-service level, teaching educator preparation programs can support equity by embedding culturally responsive pedagogy and inclusive instructional practices. Programs should offer pre-service teachers observation cycles and student teaching placements in diverse school settings. By prioritizing equity, Illinois can reduce achievement gaps and foster mathematics classrooms where every student not only belongs but contributes to learning.

## Focus of Professional Learning

Professional learning in mathematics is intentional, comprehensive, and aligned to the developmental trajectory of numeracy as described in the Framework for the Evidence-Based Development of Numeracy Skills. Educators will build numeracy in students by first strengthening their own knowledge, dispositions, and instructional practices. High-quality professional learning develops habits of mind, confidence, and evidence-based practices that bring mathematics to life.<sup>8</sup>

This focus extends beyond content. Professional learning builds teacher identity, advances equity, and connects mathematics to authentic contexts. Teachers benefit when they experience mathematics as learners by engaging in problem solving, reasoning, modeling, and discourse; those experiences translate into instruction that elicits and extends student thinking.<sup>9</sup>

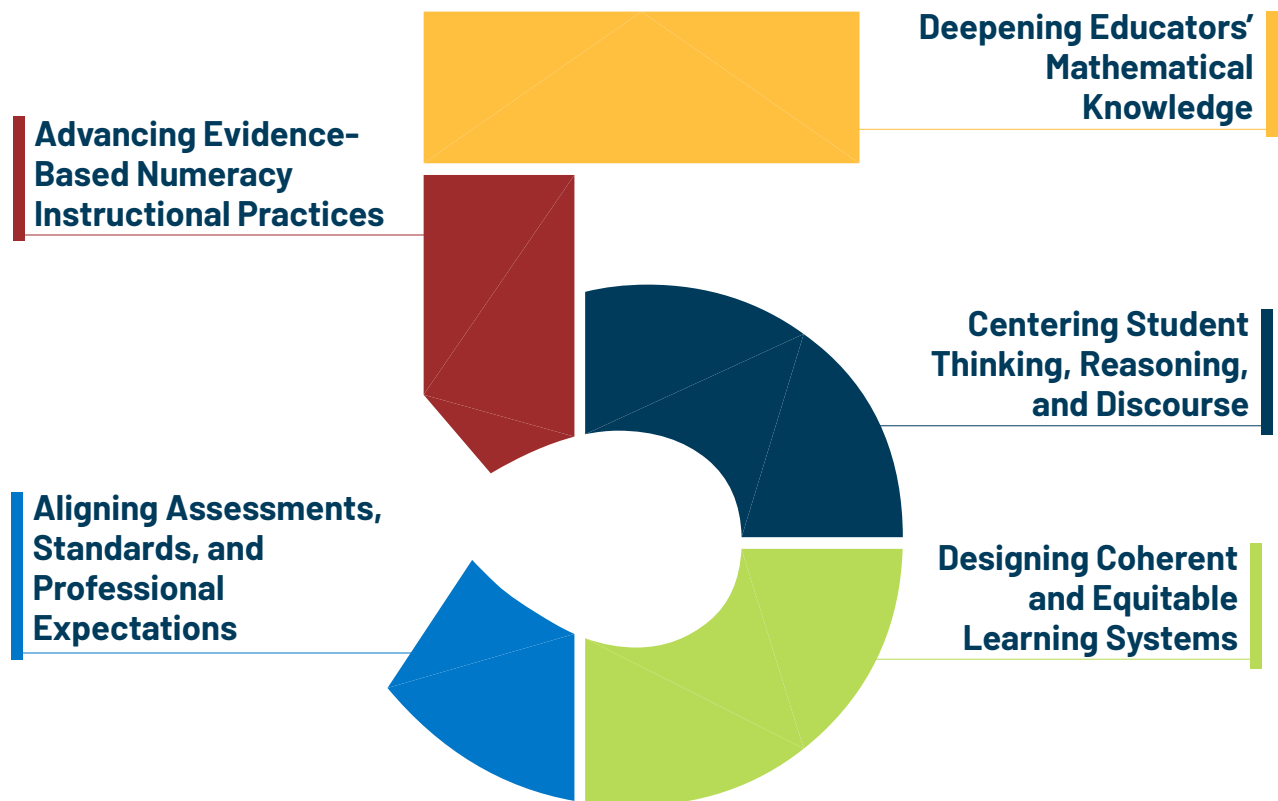
The following pages outline five interrelated domains of professional learning. Each domain includes descriptions of why it matters, what it looks like in practice, and how leaders can support its implementation. These examples are grounded in high-quality professional learning practices but are not intended to represent a single model or approach.



Professional learning should be responsive and dynamic. The needs of educators will vary based on experience, context, and student data, and may shift from year to year. Leaders must know their staff well enough to identify strengths and areas for growth, using that understanding to prioritize, differentiate, and adapt professional learning opportunities accordingly.

These domains are not exhaustive and do not encompass all knowledge and skills educators may need. Rather, they provide a coherent starting point for designing professional learning systems that are aligned to evidence-based mathematics instruction. Professional judgment remains essential in determining how best to support educator growth and, ultimately, student learning.

Five interrelated domains of professional learning:



These domains guide growth across the career continuum and align with the Illinois Learning Standards and national expectations for effective mathematics teaching.<sup>10</sup>

High-quality professional learning mirrors the same principles of strong mathematics instruction. It is content-focused, engaging teachers in mathematical ideas and pedagogy; active, providing educators and pre-service educators practice with tools, manipulatives, and strategies; collaborative, creating structures for teachers to learn from and with one another; and sustained, offering ongoing, sequential opportunities for growth.<sup>11</sup> Just as students need repeated practice to master numeracy, teachers need time and multiple experiences to master effective numeracy instruction.

## Deepening Educators' Mathematical Knowledge

Teachers must first have strong mathematical reasoning themselves before they can support students in developing it. This understanding includes deepening knowledge of numbers and operations, algebra, geometry, measurement, statistics, and probability.<sup>12</sup> Professional learning should allow educators to explore learning progressions, cross-domain connections, and fluent use of representations, such as manipulatives, visuals, symbols, and real-world contexts. Strong learning systems also nurture teacher confidence and positive mathematics identity.<sup>13</sup> Alignment to endorsement requirements in Illinois administrative code and to Illinois Licensure Testing System (ILTS) frameworks ensures coherence between teacher development and state standards.



### Why It Matters

Section 1 emphasizes that numeracy develops through a coherent progression of ideas and the integration of conceptual understanding and procedural fluency, requiring educators to understand not only grade-level standards but how mathematical ideas develop across grade bands and why procedures work. Because early gaps in number sense can compound over time, educators must understand how foundational skills develop to prevent long-term difficulties. This requires more than general content knowledge; educators must develop mathematical knowledge for teaching (MKT) to explain concepts, interpret student thinking and errors, select effective representations, and sequence instruction to build on students' understanding. Research shows that MKT predicts student achievement beyond general content knowledge alone and must be developed through targeted professional learning.<sup>14</sup> Strengthening MKT helps educators avoid practices such as teaching "tricks," premature abstraction, or overreliance on procedures, and instead supports deep, connected, and meaningful mathematical understanding.



## Deepening Educators' Mathematical Knowledge

### Teacher Actions

- ✓ Engage in math tasks across concrete-pictorial-abstract progressions
- ✓ Study learning progressions across grade bands
- ✓ Analyze how conceptual understanding and procedural fluency build upon one another
- ✓ Explore multiple representations and models as learners
- ✓ Analyze common misconceptions and plan intentional support strategies
- ✓ Identify common early numeracy difficulties and link them to underlying conceptual gaps rather than surface-level errors
- ✓ Participate in professional learning anchored to school/district-adopted high-quality instructional materials (HQIM)
- ✓ Explore how different representations, language supports, and models can increase access to mathematical concepts for MLs, students with disabilities, and students with dyscalculia while also extending thinking for advanced learners

### Leadership Actions

- ✓ Prioritize content-focused professional learning
- ✓ Build structures/procedures that promote vertical collaboration and common planning time
- ✓ Provide access to HQIM and professional resources
- ✓ Use tools such as coherence maps and curriculum evaluation tools to anchor learning in standards and maintain coherence and rigor
- ✓ Establish and sustain PLCs with shared goals, ensuring educators collaboratively deepen mathematical understanding and analyze student thinking
- ✓ Ensure professional learning includes a focus on how content knowledge connects to diverse learning needs, including language development, cognitive processing differences, and opportunities for extension

### Domain 1 in Practice:

- ✓ A district hires a math coach to lead a summer institute for K–5 teachers. Rather than reviewing curriculum pacing, teachers spend two days doing mathematics aligned to their adopted HQIM: solving problems, using manipulatives, and discussing why procedures work the way they do. The goal is not to cover content but to shift how teachers experience mathematics and to establish ongoing PLC structures where that learning continues into the school year.

## Advancing Evidence-Based Numeracy Instructional Practices



Evidence-based instructional practices refer to instructional approaches supported by rigorous, peer-reviewed research demonstrating a positive impact on student learning. Teachers need access to instructional strategies that connect fluency with understanding and promote reasoning. Professional learning should focus on fluency as a progression that moves from counting to deriving to mastery and builds on evidence-based strategies.<sup>15</sup> Educators should experience and enact proven routines like number talks, problem strings, and discourse structures that highlight student reasoning and multiple strategies to solve rich tasks throughout the K-12 continuum.<sup>16</sup> These practices must always prioritize equity, with explicit integration of the [Culturally Responsive Teaching and Learning Standards](#) and UDL so that every student can access rigorous mathematics.<sup>17</sup>



### Why It Matters

Section 1 defines evidence-based instruction and recognizes that high-quality mathematics instruction draws on a range of evidence-based instructional approaches. Evidence-based instructional practices ensure decisions and methods are grounded in research rather than trends or isolated practices. They promote consistency in instructional quality across classrooms, reducing variability in student learning experiences. Evidence-based instruction must be implemented in ways that ensure all students can access rigorous mathematics, recognizing that students may require different entry points, supports, or extensions.



## Advancing Evidence-Based Numeracy Instructional Practices

### Teacher Actions

- ✓ Experience and analyze instructional routines
- ✓ Engage in and plan for retrieval practice, spaced practice, and worked examples
- ✓ Compare instructional approaches to determine when and why specific strategies are effective
- ✓ Practice the use of formative questioning and feedback to support reasoning and fluency
- ✓ Practice instructional responses when students do not meet benchmarks (e.g., adjusting instruction, increasing opportunities for practice, targeted small group support)
- ✓ Apply UDL and CRTL principles to design instruction that supports MLs, students with disabilities, and students with dyscalculia while maintaining rigor and providing

### Leadership Actions

- ✓ Provide professional learning on evidence-based differentiation and scaffolding, ensuring supports increase access without lowering expectations
- ✓ Provide coaching cycles focused on instructional practices
- ✓ Use walkthrough tools aligned to the Standards for Mathematical Practice and Six Components of Numeracy
- ✓ Establish clear expectations for evidence-based instruction

### Domain 2 in Practice:

- ✓ A school builds number talks into its weekly PLC agenda. Teachers facilitate a number talk with their class, bring student response data to the PLC, and debrief together: What strategies came up? Who participated and who did not? What would we change? Over time, administrators ensure this cycle is sustained through coaching and protected collaboration time so teachers refine their use of evidence-based practices.

## Centering Student Thinking, Reasoning, and Discourse



High-quality professional learning positions student thinking at the center of mathematics instruction. Teachers study how mathematical strategies develop from early counting through proportional reasoning and functional thinking.<sup>18</sup> They practice eliciting and extending student ideas, orchestrating classroom discussions, and analyzing student work to guide next steps.<sup>19</sup> Professional learning also helps teachers affirm students' mathematical identities and foster agency, persistence, and productive struggle as essential elements of learning.<sup>20</sup>



### Why It Matters

Section 1 positions student thinking as the driver of instruction, emphasizing reasoning and discourse. By centering student thinking, students are positioned as active participants in learning, which can lead to increased engagement and ownership. Centering student thinking requires recognizing that students express understanding in different ways and that language, identity, and learning differences influence how students engage in reasoning and discourse. Prioritizing reasoning and discourse strengthens students' ability to transfer mathematical thinking across contexts by emphasizing explanation and justification.



## Centering Student Thinking, Reasoning, and Discourse

### Teacher Actions

- ✓ Analyze student work for reasoning, not just accuracy
- ✓ Plan for facilitation of discourse
- ✓ Practice techniques for eliciting and extending student thinking
- ✓ Experience and plan for low-floor, high-ceiling tasks
- ✓ Rehearse teacher moves that promote student-to-student discourse
- ✓ Examine how math anxiety and identity impact student participation and engagement
- ✓ Practice strategies that support participation in discourse for MLs (e.g., sentence frames, visual supports), students with disabilities, and students with dyscalculia while also encouraging advanced learners to justify and extend their thinking

### Leadership Actions

- ✓ Normalize use of student work protocols in PLCs
- ✓ Adopt observation tools that look for student thinking
- ✓ Promote grading and assessment practices that value reasoning and explanation
- ✓ Allow teachers to plan and rehearse discourse routines
- ✓ Support classroom environments and professional learning that promote inclusive discourse, ensuring all students have opportunities to contribute and be heard

### Domain 3 in Practice:

- ✓ A math coach brings copies of student work from three classrooms to a vertical team meeting. The team uses a structured protocol to sort the work by what it reveals about student understanding, not by correctness, and identifies instructional next steps. Teachers leave with a clearer picture of how reasoning develops across grade levels, while leaders reinforce expectations that PLC time centers on analyzing student thinking.

## Designing Coherent and Equitable Learning Systems



Teachers benefit from professional learning that frames mathematics as an interconnected system. Educators should explore how standards progress across grade levels, study cross-domain connections using tools like Achieve the Core's Coherence Map, and connect mathematics to other disciplines such as science, social science, and career and technical education. These experiences strengthen the ability to design instruction that emphasizes real-world application and reflects the numeracy skills students will need as adults.<sup>21</sup>



### Why It Matters

Section 1 emphasizes that numeracy develops through a cohesive continuum rather than isolated lessons. Instructional coherence reduces fragmentation in instruction, ensuring mathematics is experienced as connected and cumulative and promotes equitable access by removing systemic barriers to rigorous mathematics instruction. This includes establishing clear systems for early identification of students at risk for numeracy difficulties. Without clear systems for early identification and response, gaps widen and become more difficult to address over time.



## Designing Coherent and Equitable Learning Systems

### Teacher Actions

- ✓ Study vertical and horizontal alignment of standards and practices
- ✓ Design lessons and systems that provide multiple entry points and pathways, including scaffolds, targeted supports, and extensions, without removing students from grade-level content
- ✓ Analyze barriers to access in tasks and materials
- ✓ Plan for scaffolding and differentiation
- ✓ Collaborate across roles and departments
- ✓ Design and refine tasks to maintain rigor while increasing accessibility

### Leadership Actions

- ✓ Ensure professional learning includes specialized learning strategies
- ✓ Audit systems for equitable access to grade-level mathematics
- ✓ Conduct audits of curriculum, instruction, and intervention systems for coherence
- ✓ Ensure structures for ongoing collaboration between educators and support staff
- ✓ Define and communicate grade-level benchmarks and align supports when students do not meet expectations
- ✓ Ensure systems are in place for inclusive access, including co-teaching, targeted intervention, and enrichment, so that all students engage in meaningful mathematics

### Domain 4 in Practice:

- ✓ A math coach brings copies of student work from three classrooms to a vertical team meeting. The team uses a structured protocol to sort the work by what it reveals about student understanding, not by correctness, and identifies instructional next steps. Teachers leave with a clearer picture of how reasoning develops across grade levels, while leaders reinforce expectations that PLC time centers on analyzing student thinking.

## Aligning Assessments, Standards, and Professional Expectations



Professional learning must anchor teacher growth in state and national standards. Teachers need explicit opportunities to align their instruction to the Illinois Learning Standards for Mathematics and Standards for Mathematical Practice. Preparation and in-service learning should also reflect national expectations, such as the Association of Mathematics Teacher Educators (AMTE) Standards.<sup>22</sup> For secondary teachers, alignment includes building the capacity to connect instruction to assessments that measure college and career readiness.<sup>23</sup>



### Why It Matters

Section 1 emphasizes that assessment is only meaningful when it informs and changes instructional decision-making; its purpose is not simply to collect data but to guide what happens next in teaching and learning. When used effectively, assessment becomes an ongoing process that drives responsive instruction, shaping decisions about pacing, differentiation, intervention, and ensuring all students have access to meaningful mathematical learning.



## Aligning Assessments, Standards, and Professional Expectations

### Teacher Actions

- ✓ Design formative and summative assessments
- ✓ Use assessment data to inform instructional decisions
- ✓ Design student self-assessments and reflection tasks
- ✓ Study alignment between standards, SMPs, and Six Components of Numeracy
- ✓ Analyze assessment data with attention to student subgroups, identifying patterns in performance for MLs, students with disabilities, students with dyscalculia, and advanced learners to inform instruction

### Leadership Actions

- ✓ Provide professional learning on analyzing student thinking through data
- ✓ Align district assessments to standards and instructional priorities
- ✓ Provide structures for regular data analysis and instructional adjustment
- ✓ Ensure assessment practices and data use include equity-focused analysis, leading to instructional decisions that support access, intervention, and extension for all learners

### Domain 5 in Practice:

- ✓ An instructional coach meets monthly with the secondary math team to review assessment data against the Illinois Learning Standards for Mathematics. The team identifies which Standards for Mathematical Practice are evident in student work and which are missing, then adjusts tasks and instruction accordingly. This analysis drives the next professional learning cycle, supported by leadership through consistent expectations and time for data-informed collaboration.

## School and District Leaders

To support student learning of numeracy and teachers' evidence-based mathematics instruction, school and district leaders must act as instructional leaders. It is not enough for leaders to understand general, subject-neutral principles of learning and instruction.

When leaders cultivate their own understanding of high-quality mathematics instruction, they create the conditions for teachers and students to thrive. Leaders should engage in mathematics professional learning alongside educators, not only to strengthen their knowledge of evidence-based instructional strategies but also to signal the value of collaborative learning at every level of a system.<sup>24</sup> Leaders can also deepen their understanding by engaging in classroom observations, or "math walks," with coaches and teacher leaders, using structured "look for" tools to focus on evidence of student reasoning, teacher questioning, and equitable participation.<sup>25</sup>

Ambitious and equitable mathematics instruction requires leaders to:

- Develop a vision of high-quality numeracy in partnership with educational leaders, including teachers, community members, and students
- Promote instruction that aligns to the Illinois Learning Standards, the Standards for Mathematical Practice, and evidence-based practices
- Ensure that all teachers, regardless of content area or grade level, understand their role in building students' numeracy and actively promoting a shared, schoolwide commitment to numeracy development
- Engage in observation and feedback cycles that emphasize teacher growth, reflection, and analysis of student thinking outside and apart from formal evaluations<sup>26</sup>
- Create and sustain collaborative structures, such as PLCs, vertical teams, and data teams, where teachers analyze student work, examine progressions, and plan instruction together<sup>27</sup>
- Prioritize equity in professional learning, ensuring that teachers have access to resources, coaching, and collaborative time to meet the needs of all learners<sup>28</sup>



## Specific Learnings for Leaders

Leaders need explicit opportunities to learn how to recognize, support, and sustain ambitious and equitable numeracy instruction.

What To Look For in Classrooms	How to Provide Feedback	What Productive Collaborative Structures Look Like
<ul style="list-style-type: none"><li><b>a.</b> Evidence of student reasoning, not just correct answers</li><li><b>b.</b> Use of multiple representations (manipulatives, visuals, symbols, and real-world contexts) and models</li><li><b>c.</b> Discourse structures that promote equitable participation</li><li><b>d.</b> Connections to developmental progressions of learning standards, the Standards for Mathematical Practice, and the Six Components of Numeracy</li><li><b>e.</b> Multiple sources of data such as student work samples, formative assessments, etc., used to drive instruction</li><li><b>f.</b> Use of evidence-based instructional routines that build reasoning, fluency, and connections</li></ul>	<ul style="list-style-type: none"><li><b>a.</b> Use observation tools that are formative, non-evaluative, and tied to instructional practices</li><li><b>b.</b> Focus feedback on student learning evidence rather than teacher compliance</li><li><b>c.</b> Highlight strengths while identifying one to two specific, actionable areas for growth</li><li><b>d.</b> Engage in reflective dialogue, asking teachers to analyze student thinking and consider instructional moves</li></ul>	<ul style="list-style-type: none"><li><b>a.</b> PLCs: Teachers gather to co-plan, analyze student work, and reflect on instruction using shared protocols.</li><li><b>b.</b> Vertical Teams: Educators across grade levels examine math content progressions to strengthen coherence in numeracy.</li><li><b>c.</b> Data Teams: Teachers use multiple data sources (summative and formative) to inform instruction, focusing on growth, not deficit.</li><li><b>d.</b> Leaders provide the time, resources, and facilitation to ensure these structures remain teacher-driven and student-centered and that professional learning is sustained, ongoing, and iterative.</li></ul>

## In-Service Educators

Illinois educators need sustained professional learning across their careers to refine practice, deepen mathematical knowledge, and respond to the needs of students. In-service professional learning extends and strengthens the foundation established during educator preparation programs. Districts, schools, and Regional Offices of Education (ROEs) carry shared responsibility for ensuring that educators have equitable access to these opportunities.

In-service learning addresses areas that preparation programs cannot, such as long-term growth in mathematical knowledge for teaching, leadership development, equity-driven instructional design, and advanced assessment literacy. Effective systems recognize that professional growth develops across a career continuum and requires coherent structures that support collaboration, coaching, mentoring, and leadership opportunities.

## Core Components

These components reflect the qualities of high-quality professional learning and describe the structures Illinois districts, schools, and ROEs can use to support ongoing teacher growth. Educator learning needs vary based on experience, content knowledge, role, and context. Effective professional learning systems must be intentionally designed and calibrated to reflect these differences. Administrators play a critical role in creating the structures, supports, and differentiated opportunities necessary to ensure all educators engage in meaningful, relevant professional learning that leads to improved instructional practice.

Time and resources are critical factors in the design and implementation of professional learning, and schools and districts vary in the level of access they have to each. Effective systems recognize these constraints and prioritize high-impact, sustainable approaches. Leaders can address limitations by embedding professional learning into existing structures such as PLCs, leveraging internal expertise through teacher leadership, utilizing regional supports (e.g., ROEs or Intermediate Service Centers [ISCs]), and focusing on job-embedded practices that connect directly to daily instruction. Thoughtful use of time and resources ensures that professional learning remains meaningful and actionable, regardless of context.

### 1. Institutes and Intensives

Districts and ROEs can provide summer institutes, intersession programs, or multi-day workshops that strengthen teacher understanding of mathematics content and pedagogy. Institutes must actively engage teachers as learners of mathematics, using the same tools and routines, such as problem strings, discourse structures, and modeling, that they will later implement with students.<sup>29</sup>

### 2. Professional Learning Communities

Schools can structure Professional Learning Communities where teachers collaborate to analyze student work, unpack standards, and design coherent progressions of learning. High-functioning PLCs go beyond pacing and logistics; they include co-planning, co-teaching, peer observation, and structured reflection to build shared expertise.<sup>30</sup>



### **3. Coaching and Specialists**

Instructional coaching is a critical lever for changing practice. Math specialists and coaches provide modeling, co-planning, feedback, and cycles of observation. Coaches not only model effective strategies but also support teachers as they rehearse and refine these approaches in their own classrooms. Coaching must align to the Illinois Learning Standards and expectations for effective teaching.<sup>31</sup>

In contexts where dedicated math coaches are not available, schools can leverage alternative structures to provide similar support. This may include partnering with ROEs, ISCs, or professional organizations to access content expertise, as well as utilizing virtual coaching models, instructional networks, or shared service agreements across districts. Schools can also build internal capacity by identifying teacher leaders, facilitating peer observation and feedback cycles, and structuring PLCs to include modeling, rehearsal, and reflection.

### **4. Mentoring and Induction**

Novice teachers require structured support that bridges preparation to independent practice. Induction programs should emphasize lesson design, numeracy-rich routines, and equity-focused practices. Mentors help new teachers build confidence in discourse practices, student reasoning routines, and analysis of misconceptions.<sup>32</sup>

### **5. Online and Cross-District Networks**

Virtual platforms extend access to teachers in rural or under-resourced districts. Online professional learning communities provide video-based lesson analysis, discussion forums, and statewide collaboration.<sup>33</sup> These networks allow teachers to share practice, reflect together, and build professional community.

### **6. Partnerships**

Illinois Regional Offices of Education, higher education institutions, and mathematics-focused nonprofits and professional organizations can serve as key partners in this work. Districts are encouraged to develop local and regional partnerships aligned to their context and student population, collaborating with ROEs, universities, nonprofits, and professional associations to expand capacity, strengthen instructional expertise, and support leadership development. Effective partnerships are built through shared instructional goals, designated points of contact or committees, and ongoing communication and agreements focused on improving teaching and learning, including models such as lesson study and other collaborative improvement practices.

### **7. Sustained Engagement and Evaluation**

Professional learning must spiral across time, not occur as isolated events. Teachers need multiple opportunities to revisit ideas, apply them in practice, and refine through cycles of feedback and reflection. Districts and ROEs should monitor participation, collect teacher input, and evaluate impact using classroom observations, reflections, and student outcomes.<sup>34</sup>

Practice	When Appropriate	Alignment to Professional Standards	Less Impactful Practices
Lesson study cycles with co-planning, observation, and refinement <sup>35</sup>	When schools or districts want to build collective expertise through collaborative lesson design and reflection	Illinois Learning Standards for Mathematics; Standards for Mathematical Practice (SMP 1, SMP 3, SMP 6); Professional Standards for All Teachers, Standard 2: Content Area and Pedagogical Knowledge (23 Ill. Adm. Code 24.130(b)) [CP]	Teachers meet to discuss a lesson but do not observe or analyze student learning, leaving no actionable insights
Long-term coaching cycles that include modeling and reflection <sup>36</sup>	When districts aim to provide individualized, sustained support for teacher growth	Professional Standards for All Teachers, Standard 8: Collaborative Relationships (23 Ill. Adm. Code 24.130(h)); AMTE Standards (2017); Learning Forward Standards (2022)	A single coaching visit with generic feedback that does not include modeling, observation, or reflection
PLC protocols for analyzing student work and misconceptions <sup>37</sup>	During grade-level or department meetings where teachers collaboratively examine evidence of student thinking	Illinois Learning Standards for Mathematics; ILTS Framework performance indicators (Middle Grades 202, Secondary 208); 23 Ill. Adm. Code 26.200	PLCs that focus only on pacing guides or logistics without analyzing student reasoning or addressing misconceptions
Cross-district online communities with video libraries and discussion forums <sup>38</sup>	When educators in rural or under-resourced schools need access to professional learning networks beyond their district	Learning Forward Standards (2022); Professional Standards for All Teachers (23 Ill. Adm. Code 24.140)	Teachers access a video library but receive no structure for reflection, collaboration, or feedback



Practice	When Appropriate	Alignment to Professional Standards	Less Impactful Practices
Equity-centered workshops using culturally relevant contexts and UDL <sup>39</sup>	When schools and ROEs focus professional learning on serving multilingual learners, students with disabilities, and historically marginalized groups	CRTL Standards (23 Ill. Adm. Code 24.50); UDL Guidelines (CAST, 2018)	One-off workshops that cover equity in theory but do not connect to lesson design, practice, or student work
Mentoring structures that provide feedback on numeracy routines and discourse <sup>40</sup>	When districts design induction programs to support novice teachers in their first years	Professional Standards for All Teachers, Standard 8: Collaborative Relationships (23 Ill. Adm. Code 24.140); AMTE Standards (2017)	Assigning a mentor without structured time, focus on numeracy, or observation/feedback cycles

Figure 18: Examples of Evidence-Based Professional Learning Practices For In-Service Teachers

## Teacher Preparation

Illinois educator preparation programs are responsible for preparing candidates with mathematical knowledge, instructional practices, and professional dispositions through coursework and field experiences that form the foundation of numeracy instruction. Teacher preparation must do more than teach candidates about mathematics; it must build their numeracy as learners. Pre-service educators need opportunities to strengthen their own reasoning, build confidence through identity-affirming practices, and experience mathematics in the same active ways they will later facilitate for their students.

## Expected Outcomes at Program Completion

By the end of the program, educator preparation programs should ensure candidates are able to:

- Design and implement numeracy-rich instruction: Plan lessons that connect content to real-world and cross-disciplinary contexts, incorporate multiple representations, and select tasks that promote reasoning, problem solving, and discourse
- Elicit and extend student thinking: Use questioning strategies to surface reasoning, recognize developmental stages of numeracy, and analyze student work to address misconceptions<sup>41</sup>
- Build fluency through understanding: Facilitate number sense routines, math talks, fluency games, visual models, and intentional practice to develop foundational and derived addition and multiplication basic math facts to strengthen fluency grounded in reasoning<sup>42</sup>
- Support equitable, inclusive numeracy development: Affirm all students as mathematical thinkers through culturally and linguistically responsive pedagogy
- Engage in reflective, standards-driven practice: Align lessons with the Illinois Learning Standards and SMPs, reflect on evidence of student learning, and collaborate with colleagues in PLCs

## Coursework and Field Work Integration

Preparation programs provide a balanced structure that integrates mathematics content and pedagogical approaches practiced through fieldwork. High-quality preparation mirrors high-quality professional learning, which is content-rich, active, collaborative, and sustained. EPPs should align their curricula with the following Illinois standards and codes:

- Mathematics Standards for Elementary Teachers ([23 Ill. Adm. Code 20.120](#))
- Mathematics Standards for Mathematics Teachers in Middle Grades ([23 Ill. Adm. Code 21.150](#))
- Code of Ethics for Illinois Educators ([23 Ill. Adm. Code 22.20](#))
- Culturally Responsive Teaching and Leading Standards ([23 Ill. Adm. Code 24.50](#))

The following components reflect ISBE's guidance for high-quality educator preparation in mathematics. Programs are encouraged to review their coursework and field experiences against these components and identify areas for alignment and strengthening.

- Mathematics content courses: Content courses should emphasize reasoning, application, and connections among domains and grade bands. They model how adults use mathematics in authentic contexts and how children think about and learn these concepts.<sup>43</sup>
- Mathematics methods courses: Methods courses emphasize active learning: Candidates engage in number talks, problem strings, and inquiry-based routines to experience mathematics as learners. Faculty model effective practices and provide opportunities for candidates to design lessons that elicit and extend student thinking.
- Foundational education courses: Programs connect development, psychology, curriculum, assessment, and diversity to numeracy. Research highlights the value of integrating content with foundational knowledge to strengthen instructional decisions.<sup>44</sup>
- Fieldwork and integration: Candidates participate in a variety of fieldwork including but not limited to whole group, small group, one-on-one, and tutoring. Each setting should provide structured cycles of planning, teaching, feedback, and reflection.<sup>45</sup> Integrative experiences such as seminars and a final student teaching or equivalent experience require candidates to synthesize content, pedagogy, and equity practices into a coherent vision of numeracy instruction and demonstrate readiness for licensure.

Educator preparation programs should be organized to include mathematics content courses, mathematics methods/pedagogy courses, foundational education courses, and extensive opportunities to practice through fieldwork. Programs must align all coursework and fieldwork to the Illinois Learning Standards for Mathematics, SMPs, and Illinois endorsement requirements. Strong programs also include collaborations with local schools and actively work to recruit and support teacher candidates that reflect the diversity of Illinois' student population, thus strengthening the educator pipeline.



Practice	When Appropriate	Alignment to Professional Standards	Less Impactful Practices
Build fluency through reasoning strategies (i.e., number talks, math routines, math games, visual models, and problem strings in methods courses) <sup>46</sup>	During coursework that connects mathematical content to pedagogy, allow candidates to experience mathematics as learners	<a href="#">Illinois Learning Standards for Mathematics</a> (SMP 2, SMP 3, SMP 4); AMTE Standards (2017)	Instructor lectures about number talks without engaging candidates in the routine or analysis of student thinking
Structured rehearsal cycles in clinical placements (plan-teach-reflect-revise) <sup>47</sup>	In clinical experience or coursework where candidates practice instructional moves and receive mentor feedback	23 Ill. Adm. Code 25 (b) (Clinical Experiences); Professional Standards for All Teachers, <a href="#">CRTL Standards</a> (23 Ill. Adm. Code 24.50)	Candidate only observes or teaches a lesson once without feedback or opportunities to revise practice
Analysis protocols for student work aligned to Illinois Learning Standards <sup>48</sup>	When building assessment literacy and practicing instructional decision making based on real or sample student artifacts	Illinois Learning Standards for Mathematics; ILTS Frameworks (Middle Grades 202, Secondary 208); NCATE/CAEP Assessment Standards	Candidate reviews answer keys but does not analyze student misconceptions or reasoning
Case studies applying CRTL and UDL <sup>49</sup>	In foundations or methods courses where candidates design responses to common instructional challenges with equity at the center	Universal Design for Learning Guidelines (CAST, 2018); CRTL Standards (23 Ill. Adm. Code 24.50)	Candidate studies generic lesson plans with no consideration for multilingual learners, students with disabilities, or cultural responsiveness

Figure 19: Examples of Evidence-Based Professional Learning Practices in Teacher Preparation

## Section Summary

This section establishes the conditions necessary to bring the instructional vision of the Illinois Comprehensive Numeracy Plan to life. High-quality professional learning is intentional, sustained, and aligned to evidence-based mathematics instruction, ensuring that educators build the knowledge, skills, and dispositions needed to support all learners across the professional learning continuum. By organizing professional learning around the five domains and embedding it within coherent systems, schools and districts can create meaningful opportunities for continuous growth from preparation through advanced practice and leadership. Leadership plays a critical role in designing and sustaining these systems, ensuring that professional learning is responsive, equitable, and directly connected to classroom practice. Together, these elements define how effective professional learning can be implemented and sustained to improve mathematics teaching and learning across Illinois.

# Additional Resources

## Section 2

These resources are recommended based on feedback and insights gathered through public engagement that demonstrate strong alignment to this section of the Numeracy Plan. They provide opportunities to explore the topics in greater depth and offer additional context to support the work. Their inclusion does not constitute an endorsement by ISBE.

### Books

- *Building a Math-Positive Culture* by Cathy Seeley
- *Choosing to See: A Framework for Equity in the Math Classroom* by Kyndall Brown and Pamela Seda
- *Coaching the 5 Practices: Supporting Mathematics Teachers in Orchestrating Productive Discussions* by Bilge Yurekli, Margaret (Peg) Smith, and Mary Kay Stein
- *The Five Disciplines of PLC Leaders* by Timothy D. Kanold
- *Grading With Integrity: A Research-Based Approach Grounded in Honesty, Transparency, Accuracy, and Equity* by Douglas Fisher, Nancy Frey, and Thomas R. Guskey
- *The Illustrated Guide to Visible Learning: An Introduction to What Works Best In Schools* by John Hattie, Douglas Fisher, Nancy Frey, and John Almarode
- *The Instructional Playbook: The Missing Link for Translating Research into Practice* by Jim Knight, Ann Hoffman, Michelle Harris, and Sharon Thomas
- *Learning by Doing: A Handbook for Professional Learning Communities at Work* by Richard DuFour, Rebecca DuFour, Robert Eaker, Thomas W. Many, Mike Mattos, and Anthony Muhammad
- *Math Therapy: 5 Steps to Help Your Students Overcome Math Trauma and Build a Better Relationship With Math* by Vanessa Vakharia
- *Mathematics for Human Flourishing* by Francis Su
- *Mathematics Formative Assessment series* by Page D. Keeley and Cheryl Rose Tobey
- *Standards for Professional Learning* by Learning Forward

### Podcasts

- *The Cult of Pedagogy* hosted by Jennifer Gonzalez
- *Educating All Learners Podcast* from Educating All Learners Alliance
- *The Grading Podcast* hosted by Sharona Krinsky and Robert Bosley
- *Instructional Coaching with Ms. B* hosted by Chrissy Beltran
- *Mathematics Teacher Educator Podcast* hosted by Joel Amidon

# Additional Resources

## Section 2

### Tools

- [Achieve the Core](#)
- [Educating All Learners Alliance](#)
- [IES Practice Guides](#) and [Flyer](#)
- [Learning Forward](#)
- [Marzano Resources](#)
- [Math Observation Protocol](#)
- [Teaching Channel](#)
- [Video in the Middle](#)



## **Goal 2: Workbook**

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**Educators will use evidence-based numeracy instructional strategies to strengthen students' mathematical understanding and confidence.**

## Goal 2

Educators will use evidence-based numeracy instructional strategies to strengthen students' mathematical understanding and confidence.

### Implementation Considerations

- Professional learning should deepen mathematical content knowledge and instructional practice.
- Educators benefit from experiencing mathematics as learners through reasoning, discourse, and problem solving.
- Instructional practices should reflect evidence-based routines that support numeracy development.
- Professional learning should be ongoing, collaborative, and connected to classroom practice.
- Equity and accessibility should be embedded in all professional learning experiences.

### Notes

### Next Steps

Engage in professional learning focused on numeracy progressions and student thinking.

Apply instructional routines such as number talks, problem strings, and discourse structures.

Collaborate with colleagues to analyze student work and instructional impact.

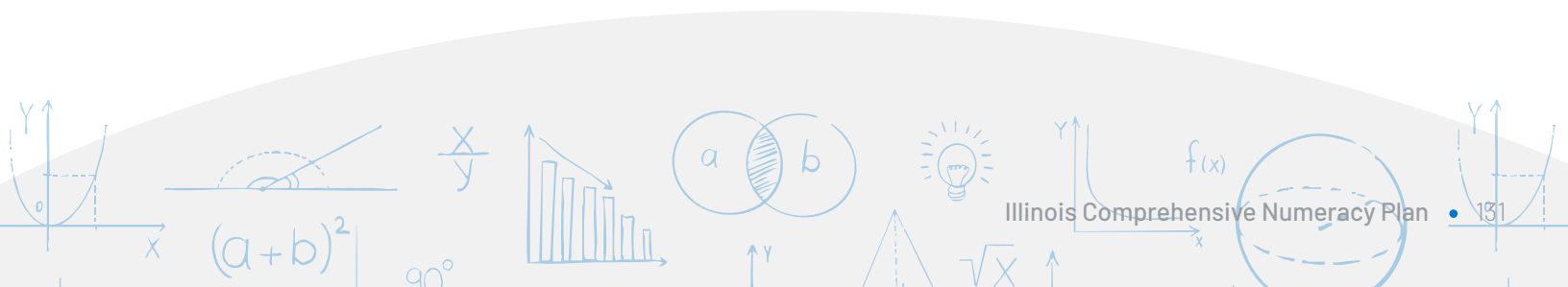
Reflect on how professional learning influences classroom practice.

Seek coaching or mentoring to refine instructional strategies.



## Reflection Questions

1. How does professional learning strengthen my understanding of numeracy development?
2. Which instructional practices have the greatest impact on student reasoning?
3. How do I apply learning from PLCs or coaching into daily instruction?
4. What additional support would help me refine my numeracy instruction?
5. How does professional learning affirm my identity as a mathematics educator?



## Goal 2

Educators will use evidence-based numeracy instructional strategies to strengthen students' mathematical understanding and confidence.

### Implementation Considerations

- Teacher leaders support sustained, job-embedded professional learning.
- Professional learning should be aligned to evidence-based numeracy instruction.
- Data and student work should guide collaborative learning.
- Coaching and mentoring are key levers for instructional improvement.
- Equity-focused practices must be reinforced through professional learning.

### Notes

### Next Steps

Facilitate PLCs focused on numeracy progressions and instructional coherence.

Support teachers in analyzing student thinking and misconceptions.

Model evidence-based instructional routines and strategies.

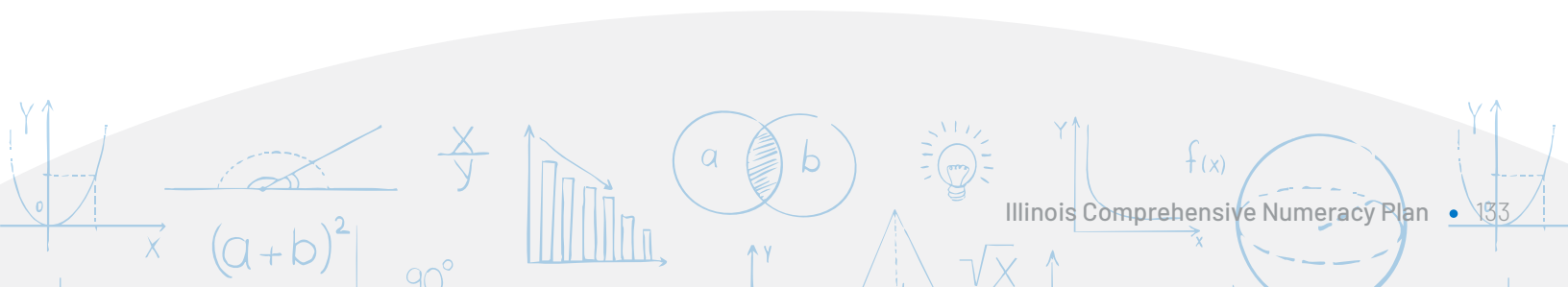
Use data to identify professional learning needs.

Collaborate with school leaders to align professional learning priorities.



## Reflection Questions

1. How do current professional learning structures support numeracy instruction?
2. How do I help teachers connect professional learning to classroom practice?
3. What patterns emerge in student work across classrooms?
4. How do I support equitable participation in professional learning?
5. What barriers limit the effectiveness of professional learning?



## Goal 2

Educators will use evidence-based numeracy instructional strategies to strengthen students' mathematical understanding and confidence.

### Implementation Considerations

- School leaders play a key role in sustaining high-quality professional learning.
- Professional learning should be aligned to schoolwide numeracy goals.
- Leaders should participate in professional learning alongside educators.
- Collaborative structures support instructional coherence and growth.
- Equity should guide decisions about access to professional learning.

### Notes

### Next Steps

Allocate time and resources for sustained numeracy-focused professional learning.

Engage in classroom observations and feedback cycles focused on student reasoning.

Support PLCs and coaching aligned to evidence-based practices.

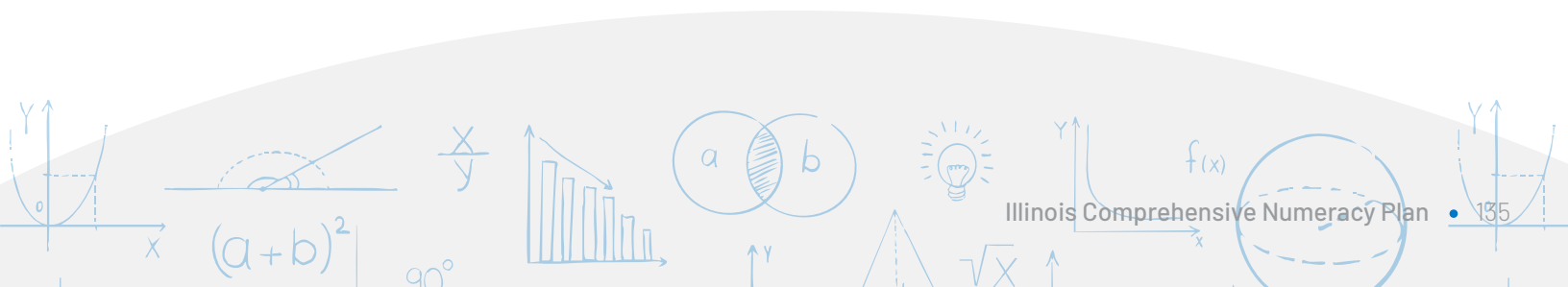
Use data to evaluate the impact of professional learning.

Promote a culture of reflection and continuous improvement.



## Reflection Questions

1. How does professional learning support schoolwide numeracy goals?
2. Are educators receiving consistent, high-quality learning opportunities?
3. How do observation and feedback practices support instructional growth?
4. How is equity addressed in professional learning decisions?
5. What structures need strengthening to sustain learning over time?



## Goal 2

Educators will use evidence-based numeracy instructional strategies to strengthen students' mathematical understanding and confidence.

### Implementation Considerations

- District systems should support professional learning across the educator continuum.
- Professional learning should be aligned to curriculum, assessment, and instruction.
- Job-embedded learning strengthens instructional coherence.
- Partnerships expand professional learning capacity.
- Equity should guide access and resource allocation.

### Notes

### Next Steps

Develop a districtwide numeracy professional learning plan.

Align professional learning with curriculum adoption and implementation.

Support coaching, mentoring, and leadership development.

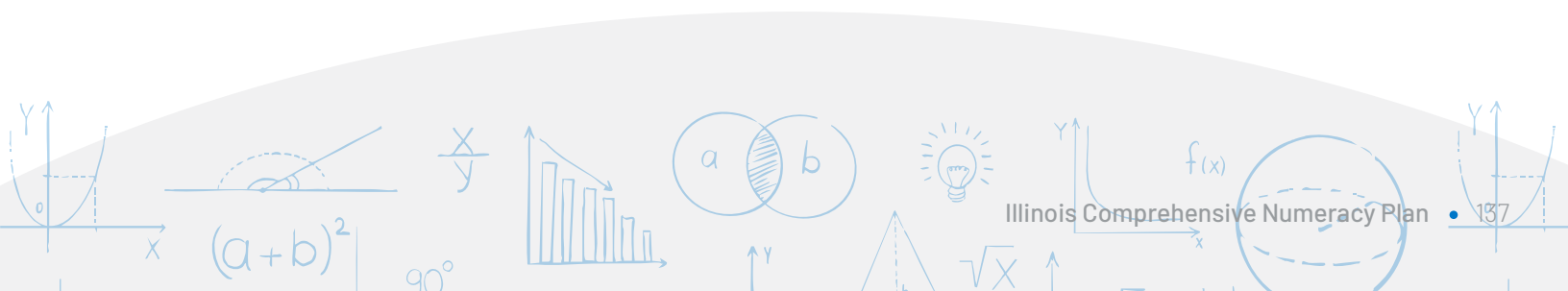
Monitor participation and impact using multiple data sources.

Address gaps in access to professional learning opportunities.



## Reflection Questions

1. How coherent is professional learning across the district?
2. What data informs professional learning priorities?
3. How are leaders supported as instructional leaders in mathematics?
4. How do partnerships strengthen professional learning systems?
5. What inequities exist in access to professional learning?



## Goal 2

Educators will use evidence-based numeracy instructional strategies to strengthen students' mathematical understanding and confidence.

### Implementation Considerations

- Regional leaders support districts through shared learning and collaboration.
- Professional learning should reflect regional needs and assets.
- Cross-district collaboration strengthens instructional capacity.
- Virtual learning expands access to professional learning.
- Equity should remain central to regional support efforts.

### Notes

### Next Steps

Conduct regional needs assessments related to numeracy instruction.

Provide professional learning aligned to evidence-based practices.

Facilitate cross-district PLCs and learning networks.

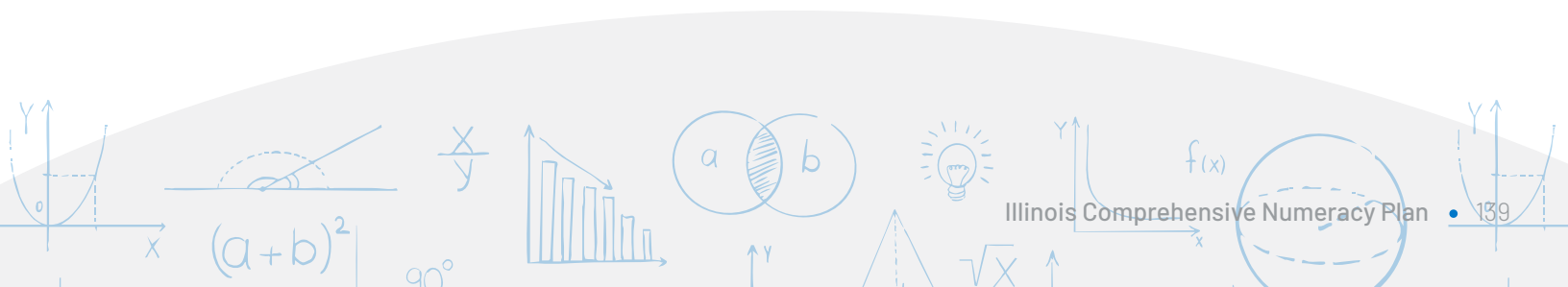
Support districts in evaluating professional learning impact.

Coordinate resources to support underserved districts.



## Reflection Questions

1. How do regional offerings align with district numeracy needs?
2. What opportunities exist for cross-district collaboration?
3. How is equity addressed in regional professional learning?
4. How effective are virtual learning structures?
5. What additional supports are needed across the region?



## Goal 2

Educators will use evidence-based numeracy instructional strategies to strengthen students' mathematical understanding and confidence.

### Implementation Considerations

- Preparation programs form the foundation of educators' numeracy knowledge.
- Coursework should integrate content, pedagogy, and equity.
- Candidates should experience evidence-based instructional practices.
- Fieldwork should reinforce numeracy-rich instruction.
- Alignment to state standards ensures coherence.

Notes

### Next Steps

Align coursework with the ICNP definition of numeracy.

Apply instructional routines such as methods courses.

Provide structured field experiences with feedback cycles.

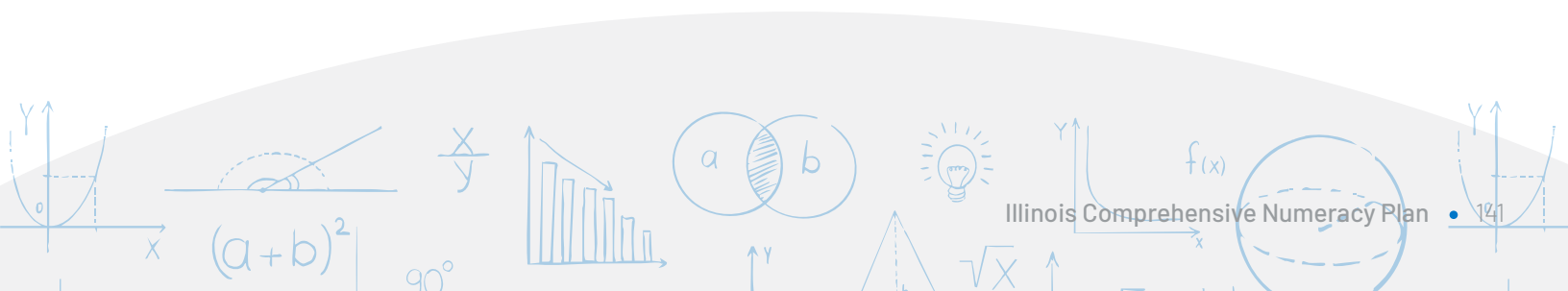
Strengthen candidate assessment literacy.

Collaborate with districts to align expectations.



## Reflection Questions

1. How do preparation programs build candidates' numeracy as learners?
2. How are candidates prepared to elicit and extend student thinking?
3. How do field experiences reflect high-quality mathematics instruction?
4. How is equity embedded in preparation experiences?
5. What gaps exist between preparation and classroom expectations?



## Goal 2

Educators will use evidence-based numeracy instructional strategies to strengthen students' mathematical understanding and confidence.

### Implementation Considerations

- State leadership shapes professional learning expectations and supports.
- Professional learning should align with statewide numeracy goals.
- Data should guide investment and improvement efforts.
- Cross-agency coordination strengthens impact.
- Equity should inform statewide professional learning strategies.

### Notes

### Next Steps

Provide statewide guidance and tools for numeracy professional learning.

Support access to high-quality professional learning across regions.

Monitor statewide participation and outcomes.

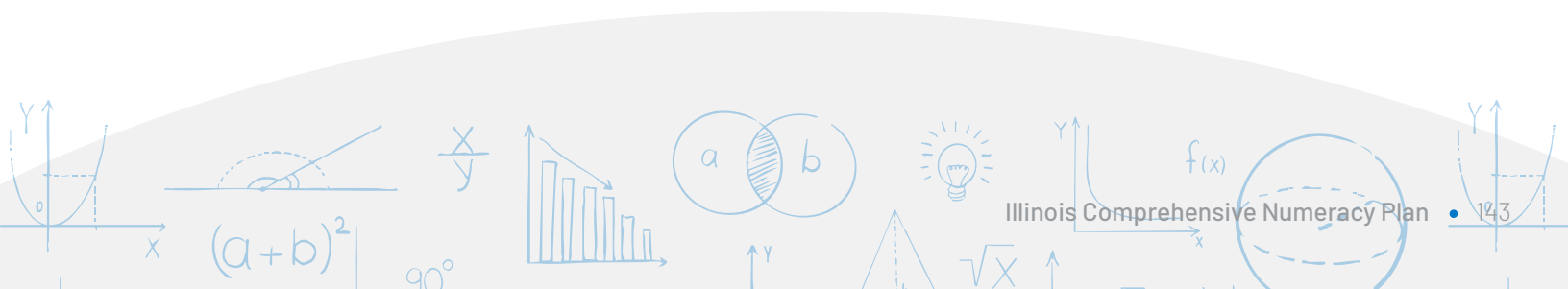
Align policy and funding to evidence-based practices.

Address systemic barriers to professional learning implementation.



## Reflection Questions

1. How does statewide professional learning support numeracy goals?
2. What trends emerge in educator participation and impact?
3. How can policy better support sustained professional learning?
4. How is equity addressed across statewide efforts?
5. What conditions are needed to scale effective practices?



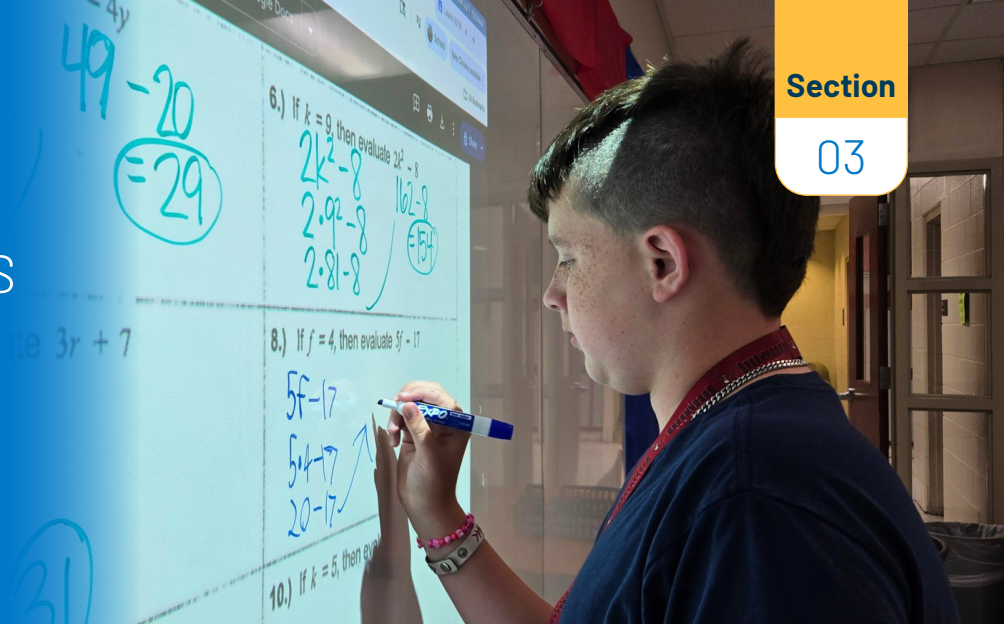
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# FRAMEWORK FOR EFFECTIVE LEADERSHIP, SYSTEMS OF SUPPORT, AND IMPLEMENTATION CONSIDERATIONS



This section is dedicated to **Goal 3**:



This section of the plan will cover the following main topics:

1. Local Framework

2. The Improvement Cycle

3. Phase 1: Initiate

4. Phase 2: Design

5. Phase 3: Unify

6. Phase 4: Implement and Monitor

7. Phase 5: Sustain and Refine



## Section Overview

This section provides a five-phase implementation framework to support districts and school leaders in developing and implementing a local plan to improve numeracy teaching and learning. ISBE strongly encourages every district and school to develop a local numeracy plan aligned to this guidance. Districts may adopt this framework directly or adapt it within their local context.

## Local Framework

A local numeracy plan is a living framework developed through shared leadership and collective inquiry to strengthen mathematics teaching and learning for all students. It aligns with and extends existing district improvement and strategic planning efforts rather than replacing them. Numeracy should be treated as a high leverage component of the district's broader goals for equity, instructional coherence, and student learning, and is not implemented in isolation.

Districts may enter the numeracy improvement framework at different phases based on their current implementation status. The phases are not a linear checklist to be completed once but a continuous improvement cycle that districts revisit as conditions evolve. Reflection questions at the beginning of each phase are intended to support districts in identifying appropriate entry points and next steps.

A local numeracy plan should reflect the district's current strengths, opportunities, and priorities, informed by multiple data sources, shared ownership, and ongoing improvement cycles already in place. A local plan does not signal fault or deficiency but represents a shared commitment to equity, evidence-based practice, and sustained professional learning while honoring existing initiatives and progress.

As student needs and contextual factors change, the plan should be refined through ongoing reflection, data use, and collaboration with educators, students, families, and community partners. Numeracy improvement is understood as an iterative, integrated process grounded in collective responsibility, trust, and coherence across initiatives, and driven by the belief that all students are capable mathematicians and all educators are valued contributors to this work.

## The Improvement Cycle

Improving student outcomes in mathematics requires intentional, systemwide leadership grounded in equity from the very beginning of the work. Building and district leaders play a critical role in strengthening numeracy instruction through establishing a clear vision, supporting coherent implementation, and ensuring that all students have access to high-quality grade-level learning experiences.<sup>1</sup> Leaders must approach mathematics improvement as an ongoing cycle of reflection, collaboration, and refinement to create the necessary structures and supports to improve numeracy rather than a one-time plan that promises a quick fix.<sup>2</sup> The creation of a local numeracy plan requires leaders to draw continuously on data and consistently engage educational partners to ensure shared ownership to increase achievement.<sup>3</sup>

In this section, leaders and teachers are specifically mentioned to delineate educator roles more clearly. To support districts and schools in systematically improving mathematics outcomes, five interconnected phases have been identified to guide planning, implementation, and continuous improvement efforts for educators:



### Phase 1: Initiate

Assist building and district leaders in analyzing, clarifying, and creating a shared vision of ownership for a local numeracy plan

- Analyze high-level numeracy data
- Identify strengths and opportunities
- Establish shared leadership
- Draft an equitable, ambitious vision for numeracy improvement



### Phase 2: Design

Collaborate with educators to create the necessary structures, systems, roles, and plans needed to create a local numeracy plan

- Convene a representative numeracy team
- Conduct deeper data analysis
- Clarify roles
- Plan professional learning
- Set benchmarks and establish conditions for success



### Phase 3: Unify

Integrate all educational partners with the design plan while prioritizing professional learning and strengthening mathematical capacity

- Convene a representative numeracy team
- Conduct deeper data analysis
- Clarify roles
- Plan professional learning
- Set benchmarks and establish conditions for success



### Phase 4: Implement & Monitor

Support teachers in the use of evidence-based instructional practices and ongoing formative and summative assessment data to monitor the plan

- Implement evidence-based practices
- Monitor real-time student data
- Celebrate small wins
- Normalize learning and missteps
- Adjust instruction based on evidence



### Phase 5: Sustain & Refine

Support educators in maintaining the long-term impact of the plan through continuous improvement cycles to sustain numeracy development and achievement

- Celebrate impact
- Scale effective practices
- Refine structures and supports
- Adapt the plan based on data, reflection, and changing conditions



## Phase 1: Initiate

The purpose of phase 1 is to assist building and district leaders in analyzing, clarifying, and creating a shared vision of ownership for a local numeracy plan.

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Phase 1  
Initiate

### Analyze Current Numeracy Data

- What high-level data is available? (State testing, benchmarking, Illinois Report Card, etc.)
- What protocols are in place to analyze data?
- Based upon the data, what are identified strengths and areas of improvement?
- To what extent do current data reveal disparities in student achievement, and how can data be disaggregated to better understand patterns across student groups, classrooms, and instructional experiences?

Effective mathematics improvement begins with a clear, data-informed understanding of the current state of instruction. Leaders must start by honestly analyzing the local landscape and acknowledging that most Illinois students are not meeting expectations in mathematics.<sup>4</sup> This work should begin with a data dive that generates actionable insights for administrators. Leaders should consider what high-level data is available such as state assessments, benchmarking, 5Essentials data, and subgroup performance data. For districts with early childhood programs, kindergarten readiness data must also be analyzed, including Kindergarten Individual Development Survey (KIDS) assessment results and Illinois Early Learning and Development Standards-aligned developmental data. Reviewing trends across grade levels, content domains, and subgroups helps identify patterns in learning, including both strengths and areas for improvement as well as possible areas of inequities.<sup>5</sup> Looking at bright spots where strong numeracy instruction is already happening can play a significant role in identifying educators who can participate in a numeracy improvement initiative. This high-level data dive also allows administrators to determine which areas will need a deeper analysis to support their local numeracy plan.

Next, administrators should reflect upon current data analysis protocols within their district or building. An effective data protocol guides the analysis process in a manner that promotes consistency, objectivity, and equity by elevating facts over judgements and interpretations. A structured approach similar to the [ATLAS Looking at Data Protocol](#) supports deeper analysis and prevents teams from focusing on solutions before fully understanding the data.

### Identify Opportunities for Shared Ownership

- Who are the numeracy experts in the building/district, and how can they be leveraged?
- What opportunities are there for vertical development within and across grade levels and content areas?
- When can time be secured to collaborate with educational partners to ensure sustainability of the plan?

For any improvement plan to be successful, shared ownership must be established early and intentionally.<sup>6</sup> Leaders should consider how they can create a system of shared ownership in which responsibility for mathematics achievement can be distributed across the building or district rather than concentrated in a single role or team. This kind of distributed leadership not only strengthens implementation by drawing on the expertise of classroom teachers, teacher leaders, instructional coaches, and administrators, but also builds resilience. When ownership of the numeracy plan is shared across multiple roles, the work is less vulnerable to disruption during leadership transitions. By genuinely leveraging educator expertise and spreading leadership responsibilities, districts create the conditions for sustained, effective improvement that can endure well beyond the tenure of any one principal or superintendent.<sup>7</sup>

Additionally, leaders should identify the math experts in their district or building such as classroom teachers, specialists, interventionists, or instructional coaches. Leveraging these individuals as model classrooms or members of a numeracy leadership team assists in building internal capacity and positions staff as partners in the work. It is important that the team includes voices representing as many impacted groups as possible and leaders with enough decision-making authority to ensure the team's ideas make it to implementation.

It is imperative that leaders secure and protect collaborative time to remove logistical barriers in support of the work and promote shared ownership. Leaders should identify multiple opportunities within the existing calendar such as early release days, institute days, late starts, staff meetings, and common planning times to support collaboration. In some cases, when possible, administrators may need to consider adjustments to the daily schedule to promote common planning times. Additionally, administrators may leverage substitute coverage to allow teachers to meet during the school day. Ultimately, early scheduling decisions can ensure collaboration and shared ownership are core components of a local numeracy improvement plan.

### **Draft a Vision**

- What are the key ingredients for numeracy success?
- What are teacher and student expectations/responsibilities?

Leaders must work collaboratively with educational partners to develop a numeracy vision that honors the principles outlined in the Illinois Comprehensive Numeracy Plan while reflecting their distinctive local context. A local plan should be equitable and ambitious, yet achievable and specific enough to guide daily decision making while remaining flexible enough to evolve based on ongoing learning and experience.

A strong numeracy vision is anchored in fairness, inclusivity, and justice, ensuring that all students can see themselves as capable mathematicians. Effective school leaders recognize that a student's ability to learn mathematics is not limited by race, gender, or other characteristics. Leaders who center equity provide adequate access to resources for all students, focus on student achievement at all learning levels, support student identity development as mathematicians, and nurture student agency.<sup>8</sup>



## Phase 2: Design

The purpose of phase 2 is for building and district leaders to collaborate with educators to create the necessary structures, systems, roles, and plans needed to create a local numeracy plan.

### Call in the Numeracy Team

- Who is involved in designing this plan, and what are their roles?
- How can the lift of this work be distributed equitably among team members?

Once time and structures are secured, the next step is to convene the numeracy team. This team should include representatives from different roles, grade levels, experience ranges, and school buildings (if applicable) to ensure diverse perspectives and broad communication networks. Consider including support staff and even students in the work. Administrators may consider faculty and staff of community colleges, universities, and local libraries and organizations as potential consultants and partners in the work to effectively support implementation of current research and ongoing professional learning related to building numeracy knowledge.<sup>9</sup> They can also be effective partners in crafting professional learning opportunities with other members of the numeracy team. ROEs and ISCs should be considered ongoing implementation partners in supporting district numeracy improvement efforts. Districts may establish review cadences with these partners, including annual plan updates and mid-year progress checks to ensure sustained alignment and support over time. Leadership may consider compensating team members for their additional responsibilities through stipends, additional planning time, or professional learning opportunities they value. It is also important to consider alternatives for funding early in the planning process in the event federal funding sources are no longer available. Community-based agencies and their potential funding resources should be included in the planning process. District leaders should consider how existing funds may be reallocated to implement the necessary components of the initiative.

Cross-district collaboration allows teacher leaders to learn from peers in different contexts while reducing the burden on any single district to develop all resources and expertise internally.<sup>10</sup> Specific opportunities for collaboration include arranging visits to schools with strong numeracy programs. Encouraging teachers to engage with professional organizations like the Illinois Council of Teachers of Mathematics, forming or participating in regional or statewide numeracy networks, and engaging support from the local Regional Office of Education can assist in establishing external partnerships.

Equally important is establishing clear roles and meeting guidelines to support efficient and equitable collaboration. Guidelines clarify expectations for participation and communication and create psychological safety to promote respectful, productive dialogue during challenging conversations.<sup>11</sup> Administrators should guide the team in distributing work equitably so that responsibilities are manageable and aligned to the strengths of individual team members.<sup>12</sup> Team composition, roles, guidelines, and workload should be thoughtfully planned to promote longevity and shared ownership.

## Dive Deeper into Data

- What data are available? (Classroom data, curriculum reviews, standards alignment, evidence-based best practices, teacher evaluations, teacher and student experience data, etc.)
- What protocols are in place to analyze data?
- Based upon the data, what are identified strengths and areas of improvement?
- To what extent are curriculum, instruction, assessment, and intervention systems coherently aligned?
- Are the instructional resources currently in use considered HQIM, and to what extent do they align to the Illinois Learning Standards and the Six Components of Numeracy?

Once the numeracy team has been established, the next step is to conduct a deeper data dive focused on classroom-level data. This includes analyzing formative and summative assessments, reviewing curriculum materials and their alignment with the Illinois Learning Standards and Six Components of Numeracy, and considering trends in teacher evaluation data.<sup>13</sup> Using established data analysis protocols ensures the team approaches the work systematically and objectively and grounds discussions in evidence. A critical component of this review is examining instructional coherence and the use of HQIM. Teams should also evaluate whether Tier 2 and Tier 3 interventions are designed as diagnostic, targeted, and increasingly intensive instruction that addresses specific skill gaps rather than simply repeating Tier 1 content. All of these elements should be evaluated to determine whether students are consistently experiencing a coherent, standards-aligned instructional system across all tiers of support.

This structured analysis allows the team to clearly identify current areas of strength and areas of improvement across grade levels and subgroups. Identified strengths should be scaled and celebrated, and areas for improvement should be approached with asset-based language and recognized as opportunities for improvement through targeted professional learning, instructional adjustments, curriculum changes, etc. At this step, the team will develop a shared understanding of instructional needs and establish the foundation for action planning.

## Create Conditions for Success

- What professional learning is needed to ensure successful implementation, monitoring, and continuous improvement of behaviors?
- What time is needed and can be leveraged to create conditions for success?
- What does success look like in year 1, year 2, year 5, etc.?
- What opportunities can be identified for caregivers and community members to be called in?
- How will all staff be informed of the plan and develop professional behaviors to support continuous improvement?
- What are some achievable short- and long-term goals that align to implementation timelines?



Creating conditions for success requires administrators and the numeracy team to intentionally plan for professional learning, defining success and involving all teachers and educational partners in the work. Schools are increasingly facing challenges that impact leaders' ability to effectively administer multiple initiatives, including teacher shortages, funding, and changing public support.<sup>14</sup> School leaders can address these issues by streamlining priorities and focusing on a few high-leverage practices that all teachers can consistently implement. The team should determine what professional learning is needed to support teachers in implementing standards-aligned planning and evidence-based best practices, mathematics intervention, MTSS, and assessment literacy. Leaders must consider what time is required for professional learning, including PLC work, instructional coaching cycles, collaborative planning, etc. It is imperative that professional learning is viewed as a sustained effort that will be embedded over time rather than a disconnected, one-time event.<sup>15</sup> When teachers are agents in shaping their professional growth, they are more likely to engage authentically in the work and less likely to undermine a change effort.<sup>16</sup>

It is important to define what success will look like over time. The team should develop clear benchmarks that reflect instructional practices and student outcomes along with short- and long-term goals. Establishing milestones helps unify efforts, measure progress, and adjust supports as needed.<sup>17</sup>

The team should identify opportunities to involve families, community members, and staff beyond the numeracy team to share the message and strengthen shared ownership. Leaders must plan how staff outside the team will be informed and engaged throughout the process of numeracy improvement. It is important for leaders to clearly communicate that this work is not a reflection of wrongdoing or poor teaching but a collective effort focused on continuous improvement and better outcomes for students. Clear communication paired with meaningful opportunities for feedback and participation ensures the plan will be supported and sustained across the school community.

## Funding and Resource Considerations

- What existing funding sources are currently supporting mathematics instruction?
- Where can current professional development, curriculum, or instructional materials budgets be redirected or better aligned to strengthen numeracy without increasing overall expenditures?
- How are ROEs and regional partnerships being leveraged to expand numeracy supports while managing costs?
- How can grants or partnerships that align with the plan's goals and timeline enhance or extend available funding?

Implementing a local numeracy plan requires intentional use of resources, but districts should not view funding as a barrier to this work. Districts are encouraged to explore multiple funding options and to align numeracy goals with existing budgets and improvement priorities.

Federal Title I, II, and IV funds may be used to support evidence-based mathematics instruction, professional learning, coaching, and instructional materials. Districts should review allowable uses of these funds to determine how current allocations might support numeracy efforts. In addition, districts may seek funding through state administered school improvement grants, local general operating or

school improvement funds, the reallocation or repurposing of existing resources, as well as partnerships with private foundations, universities, research institutions, or community organizations.

ROEs can provide professional learning and coaching related to mathematics at reduced or no cost. District leaders are also encouraged to review existing professional learning, curriculum, and materials budgets to identify opportunities to redirect resources toward numeracy priorities.

Districts with limited resources should recognize that meaningful progress can occur through small, targeted investments such as short-term coaching cycles, a one-time vertical or horizontal team meeting, or focused professional learning on high-leverage numeracy practices. These efforts can build momentum and lay the foundation for sustained numeracy improvement over time.



## Phase 3: Unify

The purpose of phase 3 is for the numeracy team to integrate all educational partners with the design plan, prioritizing professional learning and strengthening mathematical capacity.

Phase 3  
Unify

### Create Shared Ownership

- How are trust and engagement built with newcomers to finalize the vision?
- What will it look/sound like when all educational partners are engaged in continuous improvement to support this vision?

Author David Chrislip writes, “If you bring the appropriate people together in constructive ways with good information, they will create authentic visions and strategies for addressing the shared concerns of the organization or community.”<sup>18</sup> Creating shared ownership requires administrators to intentionally build trust and engagement with staff who are not part of the numeracy team as well as caregivers and community partners.<sup>19</sup> The team may consider how to show the need for change through personal stories rather than data. Personal stories should be used to effectively advocate for change. These stories connect individuals’ lived experience to wider issues and offer a compelling vision for a different future.<sup>20</sup> While data can signal the need for change in mathematics instruction, educational partners are more likely to support it when they understand current classroom struggles and can clearly see the benefits of what’s ahead.

The purpose of the work must be clearly communicated to educational partners. Fostering family engagement with numeracy is essential for developing students’ math identity. Decades of research shows that family engagement in school is one of the strongest predictors of childhood success.<sup>21</sup> Furthermore, parental attitudes toward mathematics impact students’ sense of self as a learner and the creation of strong mathematical habits of mind.<sup>22</sup> When communicating with students or caregivers, it is important to explain the work with jargon-free descriptions. Administrators can foster trust by inviting all staff to contribute ideas, acknowledge expertise, provide opportunities for engagement, and consistently follow through on commitments. Leaders also have the vantage point of multiple content areas, allowing them to see opportunities for numeracy to be intentionally embedded across the system.<sup>23</sup> This includes strengthening coherence across curriculum, instruction, assessment, and intervention systems; ensuring mathematics is reinforced in interdisciplinary learning experiences such as music, physical education, health, and career-connected learning; and supporting the development of coherent secondary mathematics pathways aligned to postsecondary and career readiness goals. These system-level connections help position numeracy as a shared responsibility rather than an isolated subject area.

Resistance to change is an expected part of school improvement, not a problem to be eliminated through increased pressure or repeated messaging. Effective leaders respond by establishing a clear, shared vision and building the conditions that enable change, including ongoing communication, opportunities for early success, and structures that support sustained implementation.<sup>24</sup> At the same time, leaders recognize that meaningful change depends on developing collective capacity, strengthening relationships, and fostering coherence across initiatives so that teachers experience change as purposeful and supported rather than fragmented.<sup>25</sup> When approached in this way, resistance becomes valuable feedback that can inform more responsive leadership actions and lead to deeper,

more sustainable implementation over time.

When shared ownership is fully realized, the entire school community works toward a common vision, though the specifics may vary based upon individual schools or districts. Systems of support related to mathematics need to nurture interactions among staff members, helping them to become instructional leaders who facilitate students' growth as mathematicians.<sup>26</sup> Generally, teachers, specialists, administrators, and support staff engage in frequent evidence-based discussions about mathematics teaching and learning. Conversations are centered on data, student work, and instructional practices and avoid assigning blame. Teams celebrate successes and collaboratively address challenges. Staff across roles and content areas take responsibility for working toward common goals. In this environment, all educational partners are aligned around the vision, contribute their expertise, and sustain momentum toward both short-term and long-term mathematics achievement goals.

### **Build Capacity**

- What tools and skills do staff feel they are effective with, and what areas have they identified for improvement?
- How can professional learning be differentiated? (e.g., grade level, competency level)
- Who can support professional learning? (e.g., team/department lead, instructional coaches, district coaches, ROE, ISBE, external partners)
- What time will be secured for this professional learning?

Building numeracy capacity in teachers begins with understanding what teachers do well and where they may need additional support. Leaders and the numeracy team can use observation data, assessment data, and teacher self-reflection to make determinations about professional learning. Professional learning should be targeted and aligned to actual classroom needs rather than generalized assumptions. Additionally, professional learning should be differentiated, providing personalized support. Differentiation can occur in small-group workshops, one-on-one coaching cycles, model lessons, peer observations, group lesson planning, etc. Administrators must identify who can support professional learning: instructional coaches, mentor teachers, department leaders, external specialists, etc.

The time in which professional learning will take place must be taken into consideration to ensure its sustainability. Opportunities could include PLC time, early release or late start days, institute days, or common plan times. Thoughtful scheduling helps teachers prioritize professional learning alongside classroom responsibilities and ensures capacity building is practical and ongoing.

### **Support Community Engagement**

- How can the “why” behind this change be communicated with families and caregivers in a jargon-free manner?
- What strategies can be shared with families and caregivers to support numeracy learning at home?
- How can mathematics messaging be incorporated into regularly scheduled school events and communication plans?
- How can growth be consistently communicated with families and caregivers in order to celebrate small wins?
- What tools and strategies can be used to promote active engagement from families and caregivers?



- How can growth be consistently communicated with families and caregivers in order to celebrate small wins?
- What tools and strategies can be used to promote active engagement from families and caregivers?

Supporting community engagement begins with clearly communicating the “why” of mathematics improvement efforts. Families are more likely to engage when they understand the purpose and goals of initiatives and how the efforts will directly benefit their child, which is best communicated when educators take the time to understand the unique needs and values of their local community.<sup>27</sup> Educators can use newsletters, school websites, and social media to ensure messaging is consistent and accessible.

Families can also be equipped with strategies to support numeracy development at home such as games, discussion prompts, and real-world mathematics applications. Families need access to models to show how to implement learning at home. They might need support when numeracy work gets challenging or is different from their school experiences.<sup>28</sup> Incorporating mathematics messaging into regularly scheduled school events, such as open houses, family nights, PTA meetings, or parent-teacher conferences, reinforces the idea that mathematics learning is a shared responsibility and encourages ongoing collaboration. Rather than focusing solely on achievement, communication with caregivers should highlight growth and effort over time and celebrate student learning milestones. Two-way communication in which caregivers can ask questions and provide insight can strengthen shared ownership and trust, ensuring the school-community partnership actively supports student numeracy growth.

Beyond communication and at-home support, schools can deepen community engagement by intentionally involving families in numeracy improvement teams and schoolwide mathematics initiatives. Families bring valuable perspectives about students’ experiences, cultural contexts, and learning strengths that can strengthen problem identification and solution design within numeracy improvement efforts. Inviting caregivers to serve on math leadership teams, participate in data review sessions, co-plan family-centered numeracy events, or provide feedback on instructional priorities fosters shared ownership of improvement goals. When families and caregivers are treated as collaborators rather than observers, numeracy initiatives are more likely to reflect community values, sustain momentum over time, and build collective responsibility for student success.<sup>29</sup>

Schools can deepen community engagement by building partnerships beyond families to include universities, local businesses, and community organizations. Schools can strengthen numeracy efforts by partnering with local universities to provide access to evidence-based practices, teacher preparation support, and opportunities for students to engage in advanced mathematical learning experiences. Local businesses and industry partners can help make mathematics relevant by connecting numeracy to real-world applications, career pathways, and problem-solving experiences aligned to workforce needs. Community organizations such as libraries, park districts, and after-school programs can reinforce numeracy development by offering accessible, informal learning opportunities that extend beyond the school day. Civic and community leaders can support numeracy initiatives by advocating for the importance of mathematics, helping align school efforts with community priorities, and expanding opportunities for authentic, community-based problem solving. These coordinated efforts help create a shared responsibility for numeracy development and support more coherent, sustained outcomes for students.

## Phase 4: Implement and Monitor

The purpose of phase 4 is to support teachers in the use of evidence-based instructional practices and ongoing formative and summative assessment data to monitor the plan.

Phase 4  
**Implement  
& Monitor**

### Continue Building Educator Capacity

- What are ways to ensure teacher proficiency in assessment literacy, data-informed decision making, and grading practices?
- How can small wins be tracked and celebrated to maintain momentum?
- How can teacher questions, missteps and misconceptions, and gradual improvement be normalized to increase professional behaviors for continuous improvement?

Ongoing attention to teacher proficiency in key instructional areas such as assessment literacy, data-informed decision making, and grading practices is paramount in the continuous improvement cycle. Administrators and the numeracy team can support teachers through targeted professional learning such as coaching cycles, collaborative planning, structured reflection, walkthroughs, lesson observations, and analysis of student work.

Small wins should be tracked and celebrated to maintain momentum and reinforce progress. This could include recognizing improved student engagement, successful implementation of new instructional strategies, and growth in mathematics domains. Celebrations can occur at staff meetings or shout-outs in newsletters. Celebrating small wins creates a culture where progress is valued and can motivate educational partners to remain in alignment with the vision.<sup>30</sup>

Equally important is normalizing teacher missteps and questions as a natural part of professional growth. When teachers feel safe to take risks, ask questions, and reflect on challenges without fear of judgment or retaliation, professional behaviors are strengthened.<sup>31</sup> This culture of trust ensures that monitoring the plan is experienced as supportive and growth-oriented rather than punitive.

### Collect, Analyze, and Monitor Real-Time Data

- What real-time data set will be consistently collected and analyzed? (Formative assessments such as exit tickets, quizzes, etc., and summative assessments such as chapter tests, unit tests, progress monitoring, benchmarking, etc.)
- What protocols can be used to analyze data in a timely manner to effectively inform instruction?
- How can feedback from families and caregivers be used to monitor and strengthen the plan?



Monitoring real-time data focuses on current, actionable information that can immediately inform instruction. Real-time data can include formative assessments, exit tickets, student work samples, classroom observations, and digital learning platform results. Students themselves are a critical source of real-time data. When students understand learning targets, monitor their progress, and set goals, they engage in metacognitive processes that strengthen self-regulation and improve mathematics learning outcomes. These practices also support persistence and provide teachers with clearer insight into gaps in student understanding.<sup>32</sup> This type of data allows educators to identify trends and gaps as they occur, rather than waiting for summative assessments or benchmark data. Real-time data protocols must be in place to rapidly collect and analyze data to make timely instructional decisions.<sup>33</sup>

Educators must actively communicate student progress with caregivers. Highlighting student growth and sharing examples of classroom strategies reinforces transparency and trust. When families understand how data informs day-to-day teaching practices, they are better equipped to support learning at home and feel connected to the improvement process.<sup>34</sup> Administrators can also continue to respond to family feedback in meaningful ways. Adjusting communication or offering additional learning opportunities based on caregiver input demonstrates family perspectives are valued.

## Phase 5: Sustain and Refine

The purpose of phase 5 is to support educators in maintaining the long-term impact of the plan through continuous improvement cycles to sustain numeracy development and achievement.

Phase 5  
**Sustain  
& Refine**

### Celebrate Impact

- What parts of this plan have been successful and can be sustained?
- How can students, teachers, leaders, and educational partners be recognized and celebrated for their work?

Celebrating impact is an essential step in sustaining improvement and reinforcing shared ownership. Administrators and the numeracy team should intentionally reflect upon which parts of the plan have been successful and are ready to be sustained or scaled. Identifying successes allows leaders to preserve effective methods of improvement while using evidence to refine areas that need continued support.

Recognition should be inclusive and intentional, honoring the contributions of students, teachers, leaders, caregivers, and educational partners. These acknowledgements can take place through assemblies, staff meetings, newsletters, social media, or community events and should reinforce that improvement is a collective effort. When celebration is embedded in the improvement cycle, it helps ensure that successful practices are sustained and that all contributors feel valued in advancing the school's vision for mathematics achievement.

### Refine Conditions for Success

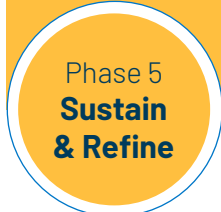
- How can continuous improvement behaviors be integrated into regular practice to avoid a one-time quick fix and be sustained for the future?
- How can this plan adapt when conditions change?
- What areas of improvement still exist, and how can these be strengthened in future cycles?
- What modifications are necessary for improvement in the next cycle?
- What tools will ensure continuous feedback and iterations of this plan?

Refining conditions for success requires recognizing that school improvement is not a one-time initiative but an ongoing, cyclical process. Administrators and teams should regularly reflect on how well the plan is functioning and remain responsive as conditions change. The plan must adapt when changes such as staffing, student needs, curriculum, or assessment expectations occur. Built-in checkpoints allow leaders to determine whether existing structures and supports should remain the same or need adjustments to sustain progress.

As part of this reflection, the numeracy team should examine where opportunities for improvement still exist. Ongoing data collection and analysis helps identify areas that require additional focus or refinement. Leaders can determine what modifications are necessary for the next improvement cycle.



To support continuous growth, administrators should utilize tools that promote ongoing feedback and iteration, including regular data reviews, reflection protocols, surveys, coaching conversations, and progress monitoring tools. These structures ensure the plan remains dynamic and responsive. Ultimately, the goal is to carry forward the lessons learned from each improvement cycle, applying insights and effective practices to future initiatives so that continuous improvement becomes embedded in the culture of the school or district.



## **Section Summary**

This section provides a five-phase implementation framework designed to guide district and building leaders in developing a coherent, equity-driven local numeracy plan. The framework positions numeracy improvement as a continuous cycle of inquiry, collaboration, and refinement rather than a linear compliance process. Across the phases, the plan emphasizes distributed leadership, the use of high-quality data protocols, and intentional collaboration with educators, families, higher education institutions, and community partners. Ultimately, the framework positions leaders as designers of coherent systems that ensure equitable access to rigorous mathematics instruction, strengthen instructional capacity across settings, and establish the conditions necessary for sustained improvement in student numeracy outcomes.

## Additional Resources

### Section 3

These resources are recommended based on feedback and insights gathered through public engagement that demonstrate strong alignment to this section of the Numeracy Plan. They provide opportunities to explore the topics in greater depth and offer additional context to support the work. Their inclusion does not constitute an endorsement by ISBE.

#### Books

- *Belonging Through a Culture of Dignity: The Keys to Successful Equity Implementation* by Floyd Cobb and John Krownapple
- *The Coaching Habit: Say Less, Ask More & Change the Way You Lead Forever* by Michael Bungay Stanier
- *Collective Leader Efficacy: Strengthening Instructional Leadership Teams* by Peter DeWitt
- *The Culture Code: The Secrets of Highly Successful Groups* by Daniel Coyle
- *The Fearless Organization: Creating Psychological Safety in the Workplace for Learning, Innovation, and Growth* by Amy Edmondson
- *Great On Their Behalf: Why School Boards Fail, How Yours Can Become Effective* by Airick Journey Crabill
- *Leadership and Self-Deception: Getting Out of the Box* by The Arbinger Institute
- *Leading for Instructional Improvement: How Successful Leaders Develop Teaching and Learning Expertise* by Stephen Fink and Anneke Markholt
- *Organizing Schools for Improvement: Lessons from Chicago* by Anthony S. Bryk, Penny Bender Sebring, Elaine Allensworth, Stuart Luppescu, and John Q. Easton
- *The Principal 50: Critical Leadership Questions for Inspiring Schoolwide Excellence* by Baruti K. Kafele
- *Street Data: A Next-Generation Model for Equity, Pedagogy, and School Transformation* by Shane Safir and Jamila Dugan
- *What Is My Value Instructionally to the Teachers I Supervise?* by Baruti K. Kafele

#### Podcasts

- *The Better Leaders Better Schools Podcast* hosted by Danny Bauer
- *The Empowered Educator Show* hosted by Dr. Mel Vandevort
- *The Learning Leader Show* hosted by Ryan Hawk
- *Principal Center Radio* hosted by Justin Baeder
- *The School Leadership Show* hosted by Mike Doughty

## Additional Resources

### Section 3

#### Tools

- [Continuous Improvement Toolkit](#)
- [Ed Instruments Math Toolkit](#)
- [Math Equity Toolkit](#)
- [TNPT Toolkits and Action Guides](#)



## **Goal 3: Workbook**

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**Leaders will promote a culture of numeracy through targeted professional learning and data-informed instructional support.**

## Goal 3

Leaders will promote a culture of numeracy through targeted professional learning and data-informed instructional support.

### Phase 1: Initiate Reflection Questions

1. What trends, strengths, and areas of need are evident in current high-level numeracy data?
2. What factors (instructional, structural, environmental) may be contributing to the numeracy outcomes reflected in the data?
3. What opportunities exist for educators, families, and community partners to engage in shared responsibility for improving numeracy achievement?
4. What characteristics should define a shared vision for effective numeracy teaching and learning in the district/building?
5. What indicators and data sources will be used to monitor progress toward numeracy goals and determine the effectiveness of implemented strategies?

Notes



## Goal 3

Leaders will promote a culture of numeracy through targeted professional learning and data-informed instructional support.

### Phase 2: Design Reflection Questions

1. What roles and perspectives should be represented on a numeracy team to ensure a balanced approach to improvement?
2. What structures will enable the numeracy team to operate effectively and sustain long-term focus?
3. What insights emerge when examining data through multiple lenses such as classroom data, curriculum reviews, standards alignment, evidence-based best practices, teacher evaluations, etc.
4. What evidence-based numeracy practices will have the greatest impact on student achievement?
5. What resources and supports are necessary to create an environment where strong numeracy instruction can be consistently implemented and continuously improved?

Notes



## Goal 3

Leaders will promote a culture of numeracy through targeted professional learning and data-informed instructional support.

### Phase 4: Implement and Monitor Reflection Questions

1. What ongoing professional learning will deepen educator capacity, when will this occur, and who will provide it?
2. What opportunities will ensure that newly developed skills and strategies are practiced, reinforced, and reflected upon?
3. What processes and tools will be used to collect, analyze, and monitor real-time numeracy data in ways that meaningfully inform instruction?

Notes



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## TOOLS AND RESOURCES



This section of the Illinois Comprehensive Numeracy Plan provides a list of tools and resources designed to support educational partners as they work to enhance numeracy instruction across the state of Illinois. The Illinois State Board of Education sets the Illinois Learning Standards for each content area and delegates the responsibility to the local school district to choose the curricular programs and instructional methods that best fit the needs of their students.

The Illinois State Board of Education does not endorse or recommend any specific curricular program or paid resources. Instead, the tools and resources provided in this section are a compilation of materials that complement the Illinois Comprehensive Numeracy Plan.

It is important to note that this section is dynamic. ISBE will continue to expand and update the list of tools and resources on the [Illinois Comprehensive Numeracy Plan webpage](#).

### A Note on Equity

Equity and inclusivity must be embedded from the start in every aspect of teaching and learning. It is important for educators and educational partners to recognize that tools and resources used for instruction should meet the needs of all learners, including those with specialized needs, multilingual learners, and gifted students. Tools should also be equitable for students at all socioeconomic levels, in all geographic locations, and across all cultural backgrounds. Therefore, ISBE encourages school districts, educators, and parents to select resources that not only align with local educational goals but also take into account the individual needs and identities of students. All students must have access to tools and resources that affirm their cultural identities, experiences, and languages.

School districts should frequently evaluate their practices, tools, and resources to ensure equity for all students and take ongoing action to promote fair and inclusive educational opportunities for all learners. [ISBE's Equity Journey Continuum](#) is a tool designed to assist districts in tracking their progress toward closing gaps in student achievement, opportunities, and supports. Districts are encouraged to utilize this tool to better view their data through a lens of equity and better support their equity journey.



**The resources listed below are organized by plan section and provide additional information, support, and guidance related to the content discussed in each section.**

## **Vision and Purpose Resources**

The federal [Every Student Succeeds Act \(ESSA\)](#) requires states to assess their learning standards for English language arts (ELA), mathematics, and science. Each state also may have a general assessment for the majority of its students and an alternate assessment for the 1% of students with the most significant cognitive disabilities. ESSA also requires that multilingual learners (MLs) be assessed in four domains (reading, writing, speaking, and listening) each year until they reach proficiency.

From the [Illinois ESSA Plan](#): “In Illinois, we believe that a universal culture of high expectations is fundamental to creating and supporting the conditions that provide the best opportunities for all students. ESSA fosters the conditions for Illinois to implement a holistic, comprehensive, and coordinated system of support that prepares each and every student for academic excellence and postsecondary success. Illinois is using the opportunities provided through ESSA to reduce barriers to learning in order to achieve fair access to high-quality educational opportunities for each and every child.” The [ISBE Assessment Department](#) oversees the assessment of students in Illinois.

The [Illinois Report Card](#) is published annually by the Illinois State Board of Education. It shows how the state as a whole, each school, and every public school district are progressing on a wide range of educational goals. The Report Card offers a complete picture of student and school performance to inform and empower families and communities as they support their local schools.

[My Data Dashboard](#) is a tool that provides administrators and teachers with detailed data related to critical performance metrics. The goal of this tool is to support data-driven decision-making and deepen understandings of how data can be used within the state, districts, schools, and classrooms.

## **Resources for Section 1: Framework for the Evidence-Based Development of Numeracy Skills**

### **Supports provided by the Illinois State Board of Education**

- The [ISBE Standards and Instruction Department](#) is committed to supporting Illinois schools and educators by providing tools, resources, and professional learning on the identification and utilization of high-quality instructional materials to increase student achievement and equitable learning outcomes. The Standards and Instruction Department also provides guidance pertaining to [advanced learners](#). Additionally, the [Illinois Learning Compass](#) is housed on the ISBE Standards and Instruction webpage. It is designed to share what all students should know and be able to do by the end of each school year. Educators, students, families, and others can use the Illinois Learning Compass to explore the Illinois Learning Standards. It allows for searching and filtering and enables the extraction of information to an Excel document.
- The ISBE [Special Education Department](#) ensures compliance with Part B of the Individuals with Disabilities Education Act (IDEA) and drives continuous improvement through Local Education Agencies. It oversees district performance to enhance educational outcomes for children with disabilities.

- The [ISBE Multilingual/Language Development Department](#) provides leadership, advocacy, and support to districts, parents, and policymakers by promoting equitable access to language support services for students from culturally and linguistically diverse backgrounds who have been identified as English learners.
- The [Early Childhood Department](#) provides leadership and technical assistance to support state programs serving children from prenatal to age 8 and their families. State Prevention Initiative, Preschool for All, and Preschool for All Expansion grants, and a variety of resources for parents, teachers, and administrators, are among services that are provided.
- The [ISBE Career and Technical Education \(CTE\) Department](#) is a dedicated team of education professionals working to provide high-quality educational programs, resources, and training for all Illinois students, teachers, and administrators. The CTE Department provides a blend of academic and CTE educational guidance, leadership, and technical assistance to local districts and regional staff that is designed to support and enhance opportunities for students to be ready for future careers.
- The [Illinois Educator Preparation Profile \(IEPP\)](#) offers a glimpse into the strength and quality of educator preparation programs across the state. The IEPP is a valuable tool for prospective educators, preK-12 administrators involved in teacher hiring, current higher education faculty and staff, parents, and others interested in learning more about educator preparation programs in Illinois. The IEPP includes program data across several key program performance indicators.

Current educators can find resources to support professional advancement and the maintenance of their Professional Educator License on the ISBE [Educator Licensure webpage](#).

### **Additional information and resources**

- The [What Works Clearinghouse](#) is an investment of the [Institute of Education Sciences \(IES\)](#) within the [U.S. Department of Education \(ED\)](#) that was established in 2002. The work of the What Works Clearinghouse is managed by a team of staff at IES and conducted under a set of contracts held by several leading firms with expertise in education, research methodology, and the dissemination of education research.
- The [Illinois MTSS Network](#) provides high-quality professional learning and coaching for schools and districts to develop and sustain a Multi-Tiered System of Supports (MTSS). MTSS is a framework for continuous improvement that is systemic, prevention-focused, and data-informed, providing a coherent continuum of supports to meet the needs of all learners.
- The [Multitiered System of Supports for English Learners](#) provides model demonstration research sponsored by the ED Office of Special Education Programs.



- [The Center: Resources for Teaching and Learning](#) is a not-for-profit organization that serves as the umbrella organization for specific programs that address different, but often related, aspects of high-quality education for students who may be at risk of academic failure. This includes students of all ages from linguistically and culturally diverse backgrounds, including multilingual learners, young children at risk of failure because of poverty, family issues, disabilities, or other circumstances; refugee and immigrant populations; and others.
- The [Self-Assessment of MTSS Implementation](#) is a needs assessment tool that helps leadership teams understand the status of MTSS implementation at the school level. The tool supports teams to engage in active discussions to identify strengths and challenges in current MTSS implementation. It is designed to help the local system identify current practices, areas of strength, and areas for growth and refinement, and enable faculty and staff to prioritize and focus resources on those areas in need of the most attention or support.
- The [Newcomer Toolkit](#), provided by ED, is a comprehensive resource designed to assist educators and community members in supporting immigrant and refugee students, referred to as multilingual learners. This toolkit is particularly valuable for state, local, and school leaders, as well as general education educators who work directly with newcomers.
- The [Dyscalculia Handbook](#), written by the [Illinois Specific Learning Disability Support Project](#), helps educators, families, and students understand dyscalculia by explaining its characteristics, impact on learning, and effective approaches to screening, data-based decision making, MTSS, and practical interventions that support improved math achievement for all students.
- [Achieve the Core](#), developed by [Student Achievement Partners](#), provides free, high-quality resources to educators. Key tools include the [Materials Alignment Toolkit](#), a suite of tools used to evaluate the alignment of instructional and assessment materials to the Shifts and major features of the Common Core, and the [Coherence Map](#), an interactive resource that shows how math standards connect within and across grade levels to support coherent instructional planning.
- The [National Council of Teachers of Mathematics](#) (NCTM) and the [Illinois Council of Teachers of Mathematics](#) (ICTM) are professional organizations focused on advancing high quality mathematics teaching and learning. NCTM is a national professional organization that supports math educators through instructional guidance, research based resources, publications, professional learning, and advocacy. ICTM is a state affiliate of NCTM that offers Illinois specific professional development, resources, conferences, and networking opportunities to support educators in implementing effective and equitable math instruction.
- The [National Association for Gifted Children](#) (NAGC) and the [Illinois Association for Gifted Children](#) (IAGC) are professional organizations dedicated to supporting the identification and education of gifted and talented children. The NAGC provides research based resources, professional learning, standards, and advocacy to advance effective gifted education nationwide. The IAGC is a state affiliate that offers Illinois specific guidance, professional development, conferences, and networking to support educators and families serving gifted learners.

Organizations that provide support to educators include:

- The [National Education Association](#) (NEA) and [Illinois Education Association](#) (IEA)
- The [American Federation of Teachers](#) (AFT) and [Illinois Federation of Teachers](#) (IFT)
  - The NEA, IEA, AFT, and IFT are labor organizations representing educators and school related professionals at the state and national levels. Together, they provide advocacy, professional learning, research, and resources to support teaching and learning, promote strong public education systems, and advance policies and working conditions that support educators, students, and communities.

## **Resources for Section 2: Educator Professional Learning and Development**

ISBE partners with stakeholders such as ROEs and ISCs to foster a robust [district and school leadership](#) pipeline that reflects the diversity of Illinois students. ISBE also supports stakeholders as they recruit, coach, and retain high-quality, equity-minded education leaders who support and reflect the diversity of students.

[Regional Offices of Education \(ROEs\) and Intermediate Service Centers \(ISCs\)](#) help ensure that every school district has a locally based point of access to numerous supports and services.

Organizations that provide support to administrators include:

- The [Illinois Principals Association](#) (IPA) provides high-quality professional development, networking opportunities, leadership support, and advocacy at the state and federal levels. Through training, collaboration, legal guidance, and access to experienced school leaders, IPA works to strengthen leadership practices that positively impact educators, students, and school communities.
- The [Illinois Association of School Administrators](#) (IASA) is a professional organization that supports superintendents and other districtlevel leaders across Illinois. IASA provides professional learning, leadership development, resources, and advocacy to strengthen district leadership and promote effective governance, instruction, and operations. Through networking opportunities and engagement with state agencies and policymakers, IASA helps school administrators address complex challenges and advance high-quality public education for all students.
- The [Illinois Association of School Business Officials](#) (IASBO) is a professional organization that supports school business leaders and district staff responsible for finance, operations, and management. IASBO provides professional learning, certification programs, resources, and networking opportunities to strengthen effective stewardship of district resources. Through training, collaboration, and advocacy, IASBO helps districts operate efficiently and sustainably in support of student learning and organizational success.
- The [Illinois Association of Regional Superintendents of Schools](#) (IARSS) is a professional organization that supports regional superintendents, assistant regional superintendents, and Regional Offices of Education across Illinois. IARSS provides leadership development, professional support, and advocacy to strengthen the delivery of educational services, influence public policy, and promote high-quality education for students and communities statewide.



- The [Association of Illinois Rural and Small Schools \(AIRSS\)](#) is the statewide organization focused on policy and advocacy for Illinois rural and small preK–12 school districts. AIRSS represents the unique needs and perspectives of rural and small schools by advocating for effective policy, legislation, and funding; elevating educator and student access to high-quality educational opportunities; and serving as a voice to state and federal policymakers to ensure the rural context is included in education discussions.
- The [Superintendents’ Commission for the Study of Demographics and Diversity](#) is a regional organization of elementary and secondary superintendents and school districts from suburban Cook and the collar counties. The commission provides high-quality educational opportunities for all students by strengthening instructional practices, supporting leadership development, and offering parent education. Through researchbased professional development, partnerships with businesses and higher education institutions, and regional seminars and workshops, the commission promotes academic achievement and equity.
- The [Illinois Alliance of Administrators of Special Education \(IAASE\)](#) is a professional organization that supports special education administrators across Illinois. IAASE provides professional learning, leadership development, resources, and advocacy to strengthen special education programs and services. The organization promotes effective practices and collaboration to improve outcomes for students with disabilities and to support the administrators who lead and manage special education systems.
- The [Ed Leaders Network \(ELN\)](#) is grounded in the belief that educational leaders impact student performance. With this foundational belief at ELN’s core, multiple state principal associations have partnered together to provide administrators with high-quality, on-demand professional development to enhance educational leadership. A dynamic professional networking community also has been created so administrators can learn and interact with their peers and leadership experts from across the country. ELN’s mission is to provide educators with the professional development and capacity-building professional network needed to do what’s best for students and the learning community.
- The [Association of Mathematics Teacher Educators \(AMTE\)](#) is a professional organization that supports the preparation and ongoing development of mathematics teachers. Its membership includes professors, researchers, teacher-leaders, school mathematics coordinators, policy experts, graduate students, and others who are focused on strengthening math teacher education from K-12.
- The [Collaboration for Effective Educator Development, Accountability, and Reform \(CEEDAR\) Center](#), in partnership with the University of Florida and the [American Institutes for Research \(AIR\)](#), supports improved outcomes for students with disabilities by strengthening educator preparation systems. CEEDAR works with states, educator preparation programs, and Local Education Agencies to build capacity for preparing teachers and leaders to implement evidence-based practices within multitiered systems of support (MTSS), with a focus on collaboration, datainformed continuous improvement, and sustainable system level reform.

## Resources for Section 3: Framework for Effective Leadership, Systems of Support, and Implementation Considerations

- ISBE’s [Curriculum Evaluation Tool](#) was designed to support best practices and continuous quality improvement, including an emphasis on equity and diversity, and the selection of high-quality instructional materials. ISBE encourages districts to use this tool to help evaluate their curriculum, foster meaningful discussions, and make decisions about the selection of new materials, as appropriate.
- [ISBE’s Family and Community Engagement Framework](#) was designed to guide the work of school and district staff to implement equitable family engagement strategies. It is also useful for families and the community at large to review expectations when engaging with their school.
- The [Prevention Initiative program](#), which is funded by the Early Childhood Block Grant, provides intensive, research-based, and comprehensive child development and family support services for expectant parents and families with children from birth to age 3 to help them build a strong foundation for learning and to prepare children for later school success.
- [Birth to Five Illinois Councils](#) support local stakeholders in coming together to identify the strengths and determine the early childhood needs within their own communities. Ensuring all children and families have access to the services they need requires a wide range of stakeholders (parents and families, school district officials, child care providers, Head Start leaders, early learning advocates, county and municipal officials, and the business community) working together in every community in Illinois to determine what families need to thrive and then creating new and enhanced services in response. Local leaders may consider utilizing Birth to Five Councils to collaborate and share the importance of early numeracy with communities and families.
- The Illinois Department of Human Services (IDHS) [Division of Early Childhood](#) administers community-based prevention and intervention programs to strengthen capacity of children, adolescents, and adults to make healthy decisions, utilize support systems, access opportunities, and achieve self-sufficiency.
- The [Large Unit District Association](#) (LUDA) is an Illinois organization representing preK–12 unit school districts with enrollments of 3,500 or more. LUDA supports district leaders through collaboration, professional learning, conferences, and advocacy. It promotes innovative educational and management practices, engages in state legislative processes, and provides timely information on policy and best practices to address the shared educational, operational, and organizational needs of large unit districts.
- The [Illinois Association of School Boards](#) (IASB) is a statewide organization that supports local school boards and district leaders. It provides guidance, professional learning, policy resources, and advocacy to help boards govern effectively and promote high-quality educational opportunities for students across Illinois.



- Overseen by IDHS, [All Our Kids Early Childhood Networks](#) (AOK Networks) promote healthy pregnancies and the positive growth and development of all children birth to 5 and their parents/caregivers by assuring a well-coordinated, easily accessible, equitable, and just system of services and supports that engages parents as partners in making the system work for them. AOK Networks use a data-driven approach to understand disparities and root causes of locally identified priority issues. The goal of the initiative is to improve outcomes for children and families through the implementation of evidence-based strategies that promote an effective local early childhood system. AOK Networks are a collaborative effort of the IDHS Division of Family and Community Services; the Illinois State Board of Education; health departments and other lead agencies representing health, early care and education, human services, and other service systems; and local stakeholders who care about the health and well-being of very young children and their parents/caregivers. It is the most comprehensive, long-standing, community-based systems development initiative in the state of Illinois.
- [Regional or local health departments](#) may also be a resource for communities.

## Glossary of Key Terms

This glossary provides definitions of terms included in the Illinois Comprehensive Numeracy Plan. Entries indicated by one asterisk (\*) were taken from the [International Literacy Association](#), and entries indicated by two asterisks (\*\*) were taken from the [Literacy Information and Communication System](#).

Term	Definition
<b>Adaptive reasoning</b>	Adaptive reasoning is the ability to think logically about the connections among concepts and situations, evaluate alternative approaches to problem solving, and justify solutions. (From <a href="#">Adding It Up: Helping Children Learn Mathematics</a> .)
<b>Assessment</b>	Assessment refers to the wide variety of methods or tools that educators use to evaluate, measure, and document the academic readiness, learning progress, skill acquisition, or educational needs of students. (From <a href="#">The Glossary of Education Reform</a> .)
<b>Automaticity</b>	Automaticity is the ability to recall facts or perform skills accurately and efficiently with little conscious effort. In mathematics, automaticity allows students to focus their attention on problem solving and reasoning rather than on basic calculations or procedures.
<b>Benchmark assessment</b>	Benchmark assessment is a process of using a screening tool multiple times across the school year to assess the effectiveness of the core curriculum and identify students at risk for failure.
<b>Cognitive load</b>	Cognitive load is the relative demand imposed by a particular task, in terms of mental resources required. (From the <a href="#">American Psychological Association</a> .)
<b>College and career readiness</b>	College and career readiness is the academic preparation that would be sufficient to allow a student to participate successfully in postsecondary education or a career without the need for remedial academic support.*
<b>Conceptual understanding</b>	Conceptual understanding is the comprehension of mathematical concepts, operations, and relations. (From <a href="#">Adding It Up: Helping Children Learn Mathematics</a> .)
<b>Culturally responsive education</b>	Culturally responsive education is the deliberate recognition and inclusion of all forms of student diversity as a pool of resources from and toward which curriculum, instruction, and all aspects of school policy should be designed. In practice, it means the alignment of curriculum and instruction with students' backgrounds, life experiences, and cultures.*
<b>Curriculum</b>	Curriculum is the overall design of instruction or opportunities provided for learning. A curriculum may include materials and textbooks, planned activities, lesson plans, lessons, and the total program of formal studies or educational experiences provided by a teacher or school. (Note: Definitions of curriculum vary widely because of alternative perceptions held by theorists about the nature and organization of formal schooling; adj. curricular.)*



Term	Definition
<b>Diagnostic screening</b>	Diagnostic screenings are administered at the beginning of the year to all students and serve to identify pre-grade and grade-level skill gaps as well as students who demonstrate readiness for acceleration.
<b>Differentiated instruction</b>	Differentiated instruction occurs when the teacher provides multiple options for learners to take information, make sense of ideas, and express what they learn. In providing diverse avenues for learners to access information, the teacher ensures that each learner learns effectively. The components of differentiated instruction include the following: (1) what to teach, or content; (2) how to teach it, or process; (3) how to find out whether learners have learned it; and (4) the environment in which the instruction occurs.**
<b>Direct instruction</b>	Direct instruction is a teacher-centered instructional approach that emphasizes the use of carefully sequenced steps that include demonstration, modeling, guided practice, and independent application. It is characterized by high rates of teacher control during the initial stages of information acquisition, followed by careful performance monitoring as the learner gradually assumes control over application. The instruction is structured, modular, and sequential (simple to complex and concrete to abstract). The teacher provides the learners with much of the information they need, often through lectures, explanations, examples, and problem solving. Most direct instruction techniques allow for only minimal learner-teacher interaction, and they need to be supplemented by review, practice, and group discussions.**
<b>Discourse structures</b>	Discourse structures are intentional routines, norms, and facilitation strategies that shape how students and teachers communicate during instruction. Discourse structures support equitable participation, clarify expectations for talk, and promote mathematical reasoning by encouraging students to explain, question, and build on one another's ideas.
<b>Dyscalculia</b>	Dyscalculia is specific learning disability affecting number sense, mathematical reasoning, and numerical processing. (From The Dyscalculia Handbook.)
<b>Equity vs. equality</b>	Equity and equality are two strategies used in an effort to produce fairness. Equity is giving everyone what they need to be successful. Equality is treating everyone the same. Equality aims to promote fairness, but it can work only if everyone starts from the same place and needs the same help.*
<b>Evidence-based practices</b>	Evidence-based practices refer to individual practices (e.g., single lessons or in-class activities) or programs (e.g., year-long curricula) supported by scientific evidence. This evidence exists within a continuum of rigor, in which some well-studied practices are highly supported while others may be promising or emerging.

Term	Definition
<b>Explicit instruction</b>	Explicit instruction is a structured, systematic approach to teaching in which the teacher clearly models concepts or skills, provides guided practice, and offers timely feedback to support accurate understanding and learning.
<b>Fluency</b>	Fluency is the ability to perform skills and use knowledge accurately, efficiently, and flexibly
<b>Formative assessment</b>	Formative assessment is the continuing study of student learning in an instructional program as it moves toward its goals and objectives by monitoring the learning progress of its participants. Diagnostic testing and various formal and informal assessment procedures can be used to identify needed adjustments to the teaching and learning activities.*
<b>Guided inquiry</b>	Guided inquiry is an instructional approach that engages students in structured exploration and critical thinking, where learners ask questions, investigate ideas, and construct understanding with intentional guidance and support from the teacher such as modeling, questioning, or scaffolding.
<b>Guided strategic development</b>	Guided strategic development is a structured form of discovery learning in which teachers use carefully sequenced, highly scaffolded instruction to guide students in developing mathematical strategies. Compared to guided inquiry, it is more intentional and more scaffolded in its design, while still preserving students' responsibility for reasoning and sense making. (From Rethinking Disability and Mathematics by Rachel Lambert)
<b>Inclusion</b>	In education, inclusion is the placement of students of all abilities in the same classroom. The term captures, in one word, an all-embracing societal ideology that involves securing opportunities for students with disabilities to learn alongside their peers without disabilities in general education classrooms.*
<b>Instructional coherence</b>	Instructional coherence is the intentional alignment of curriculum, instruction, assessments, and interventions to create a connected progression of learning that advances shared priorities and grade-level goals for all students.
<b>Intervention</b>	Intervention is additional small group or individualized instruction that is tailored to children's needs so they can make progress and be on track to meet grade-level learning goals.*
<b>Mathematical discourse</b>	Mathematical discourse is the purposeful communication of mathematical ideas, including the tools and practices that make thinking visible. It involves students articulating their reasoning, engaging with peers' ideas, and collaboratively constructing mathematical understanding.



Term	Definition
<b>Mathematical knowledge for teaching</b>	Mathematical knowledge for teaching is the specialized knowledge of mathematics that teachers use to effectively plan, teach, assess, and respond to student thinking. Mathematical knowledge for teaching includes understanding mathematical concepts, procedures, and relationships, as well as knowing how students learn mathematics, anticipate misconceptions, select representations, explain ideas clearly, and make instructional decisions that support deep mathematical understanding. (From “Effects of Teachers’ Mathematical Knowledge for Teaching on Student Achievement” by Heather Hill et al. and “Content Knowledge for Teaching: What Makes It Special?” by Deborah Loewenberg Ball et al.)
<b>Mathematics identity</b>	Mathematics identity is a person’s attitude and beliefs toward mathematics. It can be shaped by relationships, classroom experiences, cultural messages, etc.
<b>Metacognition</b>	Metacognition refers to higher-order thinking that involves active control over the cognitive processes engaged in learning: knowledge about one’s own information processing and strategies that influence one’s learning. By prompting learners to reflect on and identify the successful learning strategies that they used to solve a problem, teachers encourage learners to act on this awareness to choose appropriate learning strategies that optimize future learning. Successful learners monitor their own thought processes to decide whether they are learning effectively. Metacognitive activities include planning how to approach a given learning task, monitoring comprehension, and evaluating progress toward the completion of a task.**
<b>Multi-Tiered System of Support</b>	A Multi-Tiered System of Supports (MTSS) is a proactive and preventative framework that integrates data and instruction to maximize student achievement and support students’ social, emotional, and behavior needs from a strengths-based perspective. MTSS offers a framework for educators to engage in data-based decision making related to program improvement, high-quality instruction and intervention, social and emotional learning, and positive behavioral supports necessary to ensure positive outcomes for districts, schools, teachers, and students. See <a href="#">Multi-Tiered Systems of Support</a> .
<b>Number talk</b>	A number talk is a brief, structured classroom discussion in which students mentally solve a numerical problem and share their reasoning. Number Talks emphasize multiple strategies, mathematical reasoning, and discourse to build number sense, flexibility, and conceptual understanding.
<b>Numeracy</b>	Numeracy is the ability for all students to confidently understand, interpret, and apply mathematical concepts across all domains of mathematics in a variety of real-world and academic contexts.

Term	Definition
<b>Parallel tasks</b>	Parallel tasks are pairs or sets of related mathematical tasks designed around the same learning goal but varying in complexity or representation, allowing all students to engage in meaningful thinking while working toward a common mathematical understanding.
<b>Problem string</b>	A problem string is a purposeful sequence of related problems designed to highlight patterns, relationships, and underlying mathematical concepts. Problem strings build understanding by allowing students to connect strategies, reason flexibly, and deepen conceptual learning through carefully structured progression.
<b>Procedural fluency</b>	Procedural fluency extends beyond the ability to perform standard algorithms and involves understanding mathematical processes and procedures, knowing when and how to apply them effectively, and being able to execute them with flexibility, accuracy, and efficiency. (From <a href="#">Adding It Up: Helping Children Learn Mathematics.</a> )
<b>Productive disposition</b>	Productive disposition is a positive belief that mathematics makes sense, is valuable, and can be mastered through perseverance. It also includes confidence in students' capacity to learn and use mathematics correctly. (From <a href="#">Adding It Up: Helping Children Learn Mathematics.</a> )
<b>Professional learning</b>	Professional learning is ongoing learning provided to teachers and staff pertaining to specific strategies and skills and often based on a grade level/building/district student learning goal.*
<b>Professional learning community</b>	A professional learning community (PLC) is a collaborative group of educators who work together to improve teaching and learning. A PLC is driven by a shared focus on student learning, collective inquiry into instructional practice, the use of evidence to guide decisions, and a commitment to continuous improvement and shared responsibility for student success.
<b>Progress monitoring</b>	Progress monitoring is administered frequently throughout instruction and is aligned to the results of students' diagnostic screener. An assessment is used to determine whether students are making adequate progress toward mastery of key predecessor skills and grade-level skills gaps and to determine whether instruction should be adjusted.
<b>Retrieval practice</b>	Retrieval practice is a learning strategy that strengthens memory and understanding by having students actively recall information or skills from memory rather than re-reading or reviewing material.

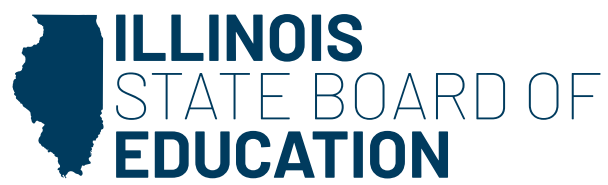


Term	Definition
<b>Scaffolded instruction</b>	Scaffolded instruction is a process that involves the frequent use of connected questions and collaboratively constructed explanations to create a context for learning based on a learner’s prior knowledge. Broad terms refer to various methods of supporting learners as they learn; gradually, these supports are withdrawn as they become capable of independent performance of a task or a skill. Supports may include clues, clarifying questions, reminders, encouragement, or breaking the problem down into steps. This temporary support from a teacher enables learners to take on and understand new material and tasks that they are not quite ready to do independently. The teacher models, assists, or provides necessary information, building on what learners already know; this should eventually lead to independence.**
<b>Standards</b>	Standards are the learning goals promulgated by a state documenting what students should know or be able to do at each grade level.
<b>Standards for Mathematical Practice</b>	The Standards for Mathematical Practice are a set of eight standards that describe the habits of mind, behaviors, and ways of thinking that students should develop to be proficient in mathematics.
<b>Strategic coaching</b>	Strategic coaching is a just in time instructional approach in which a teacher provides focused, individualized support, often one on one, to help a student develop mathematical or metacognitive strategies. Strategic coaching is responsive to what a student knows and does not yet know, offering only the level of guidance needed at the moment to move learning forward. (From Rethinking Disability and Mathematics by Rachel Lambert)
<b>Strategic competence</b>	Strategic competence is the capacity to formulate, represent, and solve mathematical problems effectively. (From <a href="#">Adding It Up: Helping Children Learn Mathematics.</a> )
<b>Summative assessment</b>	A summative assessment is the final evaluation, usually quantitative, of the degree to which the goals and objectives of a program have been attained. Different types of evidence, such as the final test score of students and the statistical analysis of program results, may enter into summative evaluation. (See formative assessment.)*
<b>Targeted professional learning</b>	Targeted professional learning is professional learning that is intentionally designed to address specific teacher needs, instructional goals, or student outcomes. It is guided by data, aligned to teachers’ roles and contexts, and focused on relevant, actionable strategies that can be applied directly to classroom practice over time.
<b>Unassisted discovery</b>	Unassisted discovery is an instructional approach in which students learn new concepts or skills through exploration with minimal guidance or scaffolding from the teacher, relying primarily on their own problem solving and inference.

Term	Definition
<b>Universal Design for Learning</b>	Universal Design for Learning is a framework for designing the educational environment so that it offers flexible learning environments that can accommodate individual learning differences. It is a key to helping all learners achieve. This environment is accomplished by simultaneously reducing or removing barriers from teaching methods and curriculum and providing rich supports for learning.**
<b>Universal screening</b>	Universal screening is the systematic assessment of all students within a given class, grade, school building, or school district on critical academic and/or social-emotional indicators.
<b>Zone of Proximal Development</b>	The Zone of Proximal Development refers to Lev Vygotsky’s “zone of readiness,” including the actions or topics a learner is ready to learn. It refers to the gap between a learner’s current and potential levels of development. This is the set of knowledge that the learner does not yet understand but has the ability to learn with guidance.**







[www.isbe.net/NumeracyPlan](http://www.isbe.net/NumeracyPlan)