

## CONFOCAL CONICS

### Performance Standard 8B.J

Write equations and graph ellipses, hyperbolas and a parabola that all have the same focus and vertex accordingly:

- *Mathematical knowledge*: know conic sections, ellipses, hyperbolas, parabolas and how to apply them in equations and graphs,
- *Strategic knowledge*: use appropriate strategies to solve the problem, and
- *Explanation*: explain completely and clearly what was done and why it was done.

### Procedures

1. Provide students with sufficient learning opportunities to develop the following skills in order to (8B) interpret and describe numerical relationships using tables, graphs, and symbols:
  - Describe the properties and features of any non-degenerate conic section from an equation or graph, and
  - Write an equation for conic sections from a graph.
  - Note: students should be familiar with conic sections but not the situation of confocal conics before attempting the assessment.
2. Provide each student a copy of the "Confocal Conics" task sheets and the rubric. Have students review and discuss the task to be completed and how the rubric will be used to evaluate it. Discussion needs to take place to be sure students understand the premise that each of the conics must have one focus at (0,1) and one vertex at (0,0). Students that have trouble understanding should make their own sketches and discuss where the other important points of each graph might be located, namely the center. They may need prompting to locate a specific center on the y-axis and then determine an equation accordingly. They will be afraid to try that to get started. It will be imperative that the student recognizes that the center of the hyperbola must be below the origin because the given vertex is the upper one as is shown in the diagram.
3. Ask students to complete the following problem and graphic:

Confocal conics are conics that share a focus. Consider the family of conics that share a focus at (0,1) and a vertex at the origin. Write equations for two different ellipses, two different hyperbolas and one parabola that have (0,0) as a vertex and (0,1) as a vertex. (Figure is on task sheet.)

4. Evaluate each student's work using the rubric as follows and its guide to determine the performance level.
  - **Mathematical Knowledge**: For a score of 4, the student must have all five equations and the graph correct and correctly labeled. There are many correct graphs, but what the student graphs and writes for an equation must match. For a score of 3, the student must have the equations nearly correct, that is with only minor computational errors on the values of "a" or "b", but appropriate values for the centers. For major computational errors on "a" or "b", score must be 2 or below.
  - **Mathematical Strategy**: For a score of 4, the student must show on the graph an appropriate choice of centers above the focus for the ellipses and below the vertex for the hyperbolas. For a score of 3, the hyperbola and ellipse centers are appropriate, but the parabola vertex is mistakenly placed somewhere besides (0,0). Inappropriate location of centers for either ellipses or hyperbolas must score 2 or below.
  - **Explanation**: For a score of 4, the student must accurately describe in writing the process for choosing the center and determining the equations from the center-to-vertex and center to focus distances, as well as explain why there is only one parabola. For a score of 3, the student must accurately or nearly accurately describe the process for choosing the centers, but may not accurately describe why there is only one parabola. Less than the requirements for a 3 must score 2 or below.

### Examples of Student Work not available

### Resources

- Copies of the "Confocal Conics" task sheets
- Mathematics Rubric

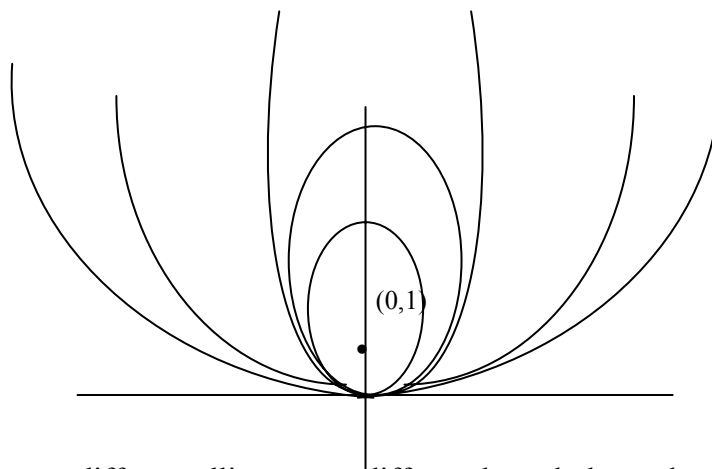
### Time Requirements

- May be done in class or assigned as a portfolio entry to be completed outside of class

NAME \_\_\_\_\_ DATE \_\_\_\_\_

### CONFOCAL CONICS - Part A

Confocal conics are conics that share a focus. Consider the family of conics that share a focus at  $(0,1)$  and a vertex at the origin. See the figure.

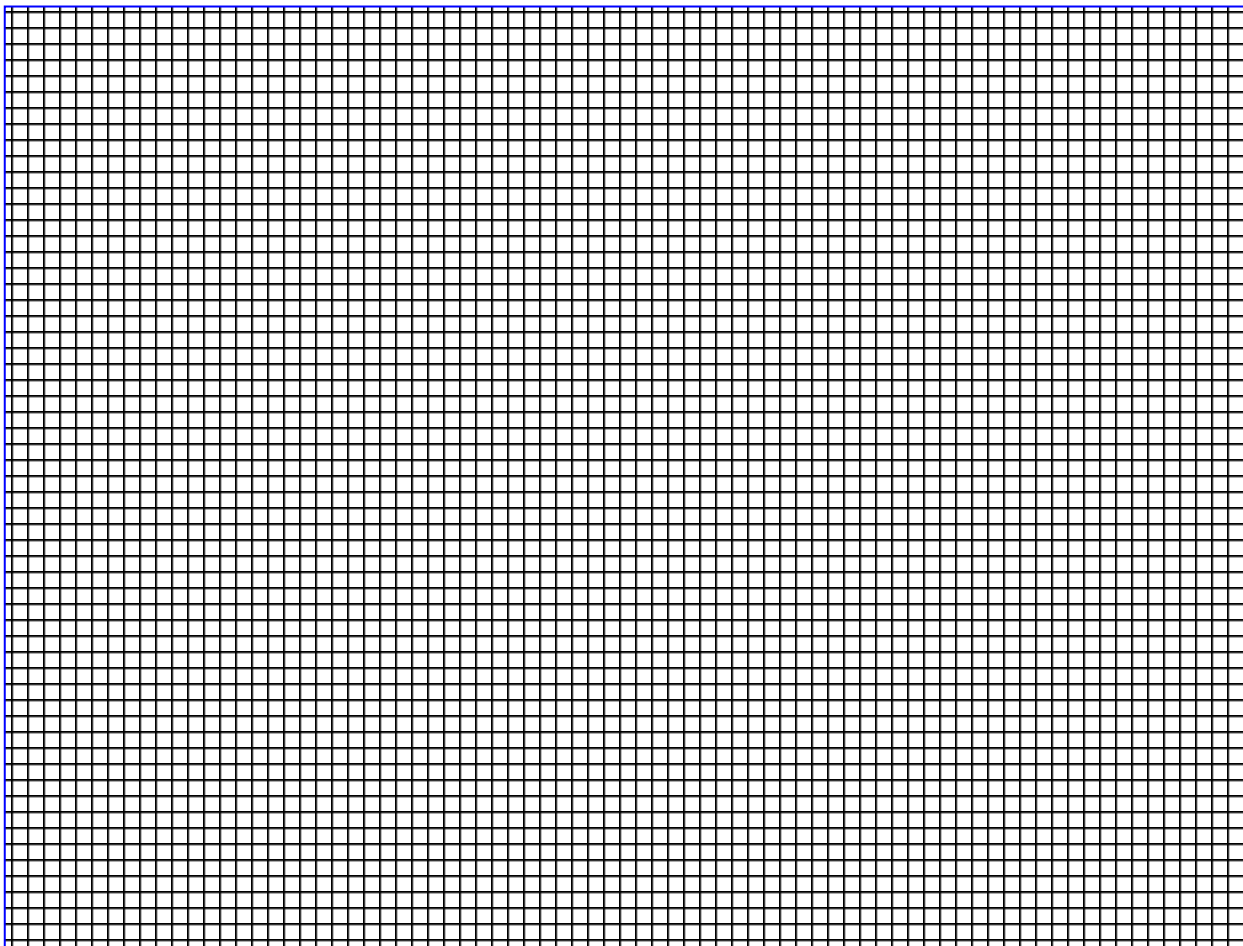


Write equations for two different ellipses, two different hyperbolas and one parabola that have  $(0,0)$  as a vertex and  $(0,1)$  as a vertex as shown in the diagram.

NAME \_\_\_\_\_ DATE \_\_\_\_\_

### CONFOCAL CONICS - Part B

Sketch below on one set of coordinate axes the conics whose equations you wrote. For the hyperbolas, sketch the top branches only. Label all centers with ordered pairs.



Explain in words the procedure you used to determine the equations, and tell why there is only one parabola with those properties.