GEARS AND WHEELS

Performance Standard (7C/9A).J

Perform calculations for linear speed and angular speed accordingly:

- *Mathematical knowledge*: Find linear and angular speed and find percent of increase.
- *Strategic knowledge*: Solve the problem.
- *Explanation*: Explain completely and clearly what was done and why it was done.

Procedures

1. ***In order to select and use appropriate technology, instruments, and formulas to solve problems, interpret results, and communicate findings (7C) and demonstrate and apply geometric concepts involving points, lines, planes, and space (9A),*** students with sufficient learning opportunities to develop the following:
   - Set up and solve measurement conversions using multiple rates and conversion factors.
   - Represent real-life objects, paths and regions in space, including intersections and cross-sections of three-dimensional figures, and describe with the language of geometry.
2. Provide each student a copy of the "Gears and Wheels" task sheet and the rubric. Have students review and discuss the task to be completed and how the rubric will be used to evaluate it.
3. Ask student to solve the following problem, show all work, and write in words what was done and why each step was done.

   The edge of a compact disc (CD) travels the length of the circumference each time it rotates through one full circle. That distance divided by time in seconds is the linear speed. If the radius is 15 cm and it turns 20 times per second, then the linear speed is $2\pi(15)(20)$ or 600π cm per second. Angular speed is defined to be the angle divided by time or Θ/t, where the angle is measured in radians. A merry-go-round making 8 revolutions per minute has an angular speed of $8(2\pi)$ divided by 1 minute which is 16π radians per minute.

   (1) Find the linear speed of a point two inches from the center of a circular gear rotating at the rate of 200 revolutions per minute (rpm); then, compare it to the linear speed of a point 2.25 inches from the center of the same gear at the same rpm.
   (2) A bicycle has wheels that are 26 inches in diameter. If the bike is traveling 14 miles per hour, what is the angular speed of each wheel?
   (3) If the bicycle decreases its speed to 12 miles per hour, find the new angular speed and determine by what percent the angular speed decreases.

4. Evaluate each student's work using all three dimensions of the rubric and its guide to determine the performance level. Minor computation errors would be an incorrect answer with correct strategy. A 4 in mathematical knowledge would require the correct answers as follows:
   - Faster by 100π per minute or 12 ½ % faster.
   - 1,786.36 radians/minute or 29.77 radians/second.
   - 45.4% decrease based on 974.77 radians/minute or 16.246 radians/second.

   A 4 in strategy would require the use of the correct formulas and conversion factors. A 4 in explanation would require a complete description of the what and why of each step.

Examples of Student Work follow

Resources

- Copies of the "Gears and Wheels" task sheet
- Calculator
- Mathematics Rubric

Time Requirements

- 25 minutes

Name ____ Date ________

GEARS AND WHEELS

The edge of a compact disc (CD) travels the length of the circumference each time it rotates through one full circle. That distance divided by time in seconds is the linear speed. If the radius is 15 cm and it turns 20 times per second, then the linear speed is $2\pi(15)(20)$ or 600π cm per second.

Angular speed is defined to be the angle divided by time or Θ/t, where the angle is measured in radians. A merry-go-round making 8 revolutions per minute has an angular speed of $8(2\pi)$ divided by 1 minute which is 16π radians per minute.

(1) Find the linear speed of a point two inches from the center of a circular gear rotating at the rate of 200 revolutions per minute (rpm); then, compare it to the linear speed of a point 2.25 inches from the center of the same gear at the same rpm.

(2) A bicycle has wheels that are 26 inches in diameter. If the bike is traveling 14 miles per hour, what is the angular speed of each wheel?

(3) If the bicycle decreases its speed to 12 miles per hour, find the new angular speed and determine by what percent the angular speed decreases.

Show all work and write in words what you did and why you did each step.

Adapted from Hungerford, Contemporary Precalculus, Orlando, Harcourt, Brace & Co., 1997, p. 375.

"Meets"

Gears and Wheels

The edge of a compact disc (CD) travels the length of the circumference each time it rotates through one full circle. That distance divided by time in seconds is the linear speed. If the radius is 15 cm and it turns 20 times per second, then the linear speed is $2\pi(15)(20)$ or 600π cm per second.

Angular speed is defined to be the angle divided by time or Θ/t, where the angle is measured in radians. A merry-go-round making 8 revolutions per minute has an angular speed of $8(2\pi)$ divided by 1 minute which is 16π radians per minute.

1. Find the linear speed of a point two inches from the center of a circular gear rotating at the rate of 200 revolutions per minute (rpm), then compare it to the linear speed of a point 2.25 inches from the center of the same gear at the same rpm. 2in 2.25
2. A bicycle has wheels that are 26 inches in diameter. If the bike is traveling 14 miles per hour, what is the angular speed of each wheel? 13 . 13 $r = 13$ in 1 mi = 5280 ft
3. If the bicycle decreases its speed to 12 miles per hour, find the new angular speed and determine by what percent the angular speed decreases.

Show all work and write in words what you did and why you did each step.

1) $2\pi(2\text{ in})(200\text{ rpm}) = 800\pi$ in per minute — I used the equation above

$2\pi(2.25)(200\text{ rpm}) = 900\pi$ in per minute — $r \times 2\pi \times$ revolutions give you (d/t); $2\pi r$ gives circumference

2) $14\frac{mi}{hr} \cdot \frac{5280 ft}{1 mi} \cdot \frac{12 in}{1 ft} = \frac{887040 \text{ in/hr}}{26\pi \text{ in/rev}} \cdot 2\pi \frac{rad}{rev} \approx 68233$ radians per hour

$2\pi \cdot 13 = 26\pi$ in/rev

I converted to find rate in in/hr. The $\div$ by circumference gives you rev/hr. Then (x) by 2π you find rad/hr.

3) $12 \text{ mi/hr} \cdot \frac{5280 ft}{1 mi} \cdot \frac{12 in}{1 ft} = \frac{760320 \text{ in/hr}}{26\pi \text{ in/rev}} \cdot 2\pi \frac{rad}{rev}$ (a) ≈ 58486 radians per hour

(b) $\frac{68233 - 58486}{68233} \times 100\% = 14.3\%$ decrease

I used the same procedure as #2 and found the % diff between the 2 #s.

Adapted from Hungerford, Contemporary Precalculus, Orlando, Harcourt, Brace & Co., 1997, p. 375.

7C9A-JM

"Exceeds"

Gears and Wheels

The edge of a compact disc (CD) travels the length of the circumference each time it rotates through one full circle. That distance divided by time in seconds is the linear speed. If the radius is 15 cm and it turns 20 times per second, then the linear speed is $2\pi(15)(20)$ or 600π cm per second.

Angular speed is defined to be the angle divided by time or Θ/t, where the angle is measured in radians. A merry-go-round making 8 revolutions per minute has an angular speed of $8(2\pi)$ divided by 1 minute which is 16π radians per minute.

1. Find the linear speed of a point two inches from the center of a circular gear rotating at the rate of 200 revolutions per minute (rpm), then compare it to the linear speed of a point 2.25 inches from the center of the same gear at the same rpm.

2. A bicycle has wheels that are 26 inches in diameter. If the bike is traveling 14 miles per hour, what is the angular speed of each wheel?

3. If the bicycle decreases its speed to 12 miles per hour, find the new angular speed and determine by what percent the angular speed decreases.

Show all work and write in words what you did and why you did each step.

1) $\frac{2in}{200rpm}$ r=2in $(2in)(200rpm)(2\pi) = 800\pi$ in per/min

$(2.25in)(200rpm)(2\pi) = 900\pi$ in per/min

I multiplied the radius by the speed it rotated, then by 2π. That gave me the linear speed. The point farther from the center traveled a greater distance than the point closer.

2) 26in=D R=13in speed $= \frac{887040in}{1hour}$ circumference $= 26\pi$

$14mile\left(\frac{63360in}{1mile}\right) = 887040$

$\frac{887040in}{26\pi} = 34116$ in/1hour

$34116(2\pi) = 68233$ rad/hr

3) 12mph R=13in speed: $\frac{760320}{1hr}$

$12\left(\frac{63360in}{1mile}\right) = 760320$

circumference = 26π

$\frac{760320}{26\pi}(2\pi) = \boxed{58486 \text{ rad/hr}}$

Adapted from Hungerford, Contemporary Precalculus, Orlando, Harcourt, Brace & Co., 1997, p. 375.

% decrease:

$\frac{21718\pi - 18267\pi}{21718\pi}$ = 14.3% decrease

$\frac{\text{original} - \text{new}}{\text{original}}$

For #2, and #3, I first converted from miles to inches, then I divided the distance by the circumference and multiplied it by 2π. That gives me the angular speed.

7C9A-JE