INVESTMENT PLANS

Performance Standard (6C/8A/8C).J

Write the explicit and recursive forms of the sequence of an investment and compare the amount to the continuously compounded interest formula for the same time period, interest and principal accordingly:

- *Mathematical knowledge*: Generalize patterns using recursively defined sequences; know how to apply sequences and their properties to solve real problems.
- *Strategic knowledge*: Solve the problem.
- *Explanation*: Explain completely and clearly what was done and why it was done.

Procedures

1. ***In order to compute and estimate using mental mathematics, paper-and-pencil methods, calculators, and computers (6C), describe numerical relationships using variables and patterns (8A), and solve problems using systems of numbers and their properties (8C),*** students should experience sufficient learning opportunities to develop the following:
   - Estimate an appropriate answer for a given term of a sequence.
   - Translate between explicit and recursive forms of sequences where possible.
   - Symbolize growth patterns using variables.
   - Generalize patterns using explicitly defined and recursively defined sequences.
   - Explain the differences and similarities of different forms of growth formulas.
   - Apply sequences and their properties to solve real problems.
2. Provide each student a copy of the "Investment Plans" task sheet and the rubric. Have students review and discuss the task to be completed and how the rubric will be used to evaluate it.
3. Ask students to do the following problem:

   Find and write the first five terms (years) of the sequence representing an investment by a 21-year-old of $2000 at 5.5% per year. Write the recursive form for that sequence. Decide what kind of sequence it is and write the explicit form of the sequence and use that to find the value of the investment at age 55. Compare the amount at age 55 from #3 to an investment at age 21 of $2000 compounded continuously at 5.5%, and describe the mathematics that makes the amounts different. Show all work and write in words what you and did and why you did each step.

4. Evaluate each student's work using all 3 dimensions of the rubric and its guide to determine the performance level. A 4 in mathematical knowledge should reflect a sequence of 2110.00, 2226.05, 2348.48, 2477.65, and 2613.92 representing amounts at the end of each year; a recursive form of $a_1 = 2110$, $a_n – a_{n-1}(1.055)$; a geometric sequence $a_n = 2000(1.055)$; $a_{34} = 12348.48$; and a continuously compounded amount of $12976.59 which is 628.11 more than periodic compounding. The mathematics involves compounding more often which results in a larger amount. A 4 in strategy would require the appropriate formulas. A 4 in explanation would require a complete explanation of the what and why of the process.

Examples of Student Work follow

Resources

- Copies of the "Investment Plans" task sheet
- Calculator
- Mathematics Rubric

Time Requirements

- 30 minutes

Name ____ Date ______________

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1. Find and write the first five terms (years) of the sequence representing an investment by a 21-year-old of $2000 at 5.5% per year.

2. Write the recursive form for the sequence in #1.

3. Decide what kind of sequence it is and write the explicit form of the sequence and use that to find the value of the investment at age 55.

4. Compare the amount at age 55 from #3 to an investment at age 21 of $2000 compounded continuously at 5.5%, and describe the mathematics that makes the amounts different.

Show all work and write an explanation of what you did <u>and</u> why you did each step.

Source: Developed for the Performance Standards Project, Illinois State Board of Education, Learning Standards Division, 2000.

"Meets" (page 1)

① To find the five terms of the sequence I took the principle ($2000) and multiplied it by 105.5% to find how much it would become after one year at a 5.5% interest rate. I then took the answer I got (2110), and multiplied it by 105.5% also, and so continued in that fashion for all five terms.

② I found the recursive form for the sequence by looking at how I built on each equation from the previous one. I saw that each time I multiplied 1.055 (105.5%), I was adding that number into the equation - so [2110 × 1.055] is actually [$2000 \times (1.055)^2$].

③ It is geometric because multiplication is done in each equation. I found the explicit form by using the principle, the percentage multiplied by, and the exponent determining how many times 1.055 was used. I then used my formula to find the investment at age 55 by subtracting 21 from 55 to find the 34 year period, and plugging in 34.

④ I used the continuous compound formula to find my answer. This time it was larger because "e" was used, which is equal to 2.718 inversely, which is greater than 1.055.

6C8A8C-JM1

① $\$2000(1.055) = 2110$

$2110(1.055) = 2226.05$

$2226.05(1.055) = 2348.48$

$2348.48(1.055) = 2477.65$

$2477.65(1.055) = 2613.92$

② $a_n = a_{(n-1)}(1.055)$

③ geometric sequence

$a_n = 2000(1.055)^n$

$a_{34} = 2000(1.055)^{34}$

$2000(6.174) = \$12348.48$

④ $B = Pe^{rt}$

$B = 2000e^{.055(34)}$

$2000e^{1.87}$

$2000(6.488) = \$12976.59$

$$\begin{array}{r} 12976.59 \\ -\ 12348.48 \\ \hline \end{array}$$

$628.11 → difference