

RIVER WIDTH

Performance Standard (6C/7B/9D).J

Use the Law of Sines and the Law of Cosines to determine the indirect measurement of the width of a river accordingly:

- *Mathematical knowledge:* Apply the Law of Sines and the Law of Cosines.
- *Strategic knowledge:* To solve the problem.
- *Explanation:* Explain completely and clearly what was done and why it was done.

Procedures

1. *In order to compute and estimate using mental mathematics, paper-and-pencil methods, calculators, and computers (6C), estimate measurements and determine acceptable levels of accuracy (7B), and use trigonometric ratios and circular functions to solve problems (9D),* students should experience sufficient learning opportunities to develop the following:
 - Determine the level of accuracy needed for computations involving measurement and irrational numbers.
 - Use the correct number of digits in computation to achieve an appropriate unit or level of accuracy when solving problems.
 - Solve problems to a desired interval of accuracy.
 - Solve problems using the Law of Sines and Law of Cosines.
2. Provide each student a copy of the "River Width" task sheet and the rubric. Have students review and discuss the task to be completed and how the rubric will be used to evaluate it.
3. Ask students to solve the following problem. Encourage students to analyze what is known about the problem, what needs to be known, and the minimum information needed to use each law.

A surveyor has been hired to find the width of the Illinois River. Survey points are located as follows: A is on one side of the river; B and C are on the other side; D is in line with AB, and E is in line with AC as shown in the figure. BC measures 506.23 feet; BD measures 453.13 feet; BE measures 809.92 feet; CD measures 753.61 feet, and CE measures 392.77 feet. Find the width of the river (from A to BC) to the nearest hundredth of a foot.

4. Discuss with students the methods of computation that do not compromise the accuracy of the final answer by rounding too soon in the procedure.
5. Evaluate each student's work using the rubric and its guide to determine the performance level. Use the standard rubric. A 4 in mathematical knowledge would require a correct answer of 495.01 feet. If the final answer is incorrect by no more than 0.2, then the score should be a 3 for mathematical knowledge. Level 4 strategy should show the use of the Law of Sines and the Law of Cosines in an easy-to-follow method and an appropriate sequence of steps. A 4 in explanation would require a complete description of the what and why of each step.

Examples of Student Work follow

Resources

- Copies of the "River Width" task sheet
- Graphing or scientific calculator
- Mathematics Rubric

Time Requirements

- One class period

ASSESSMENT (6C/7B/9D).J

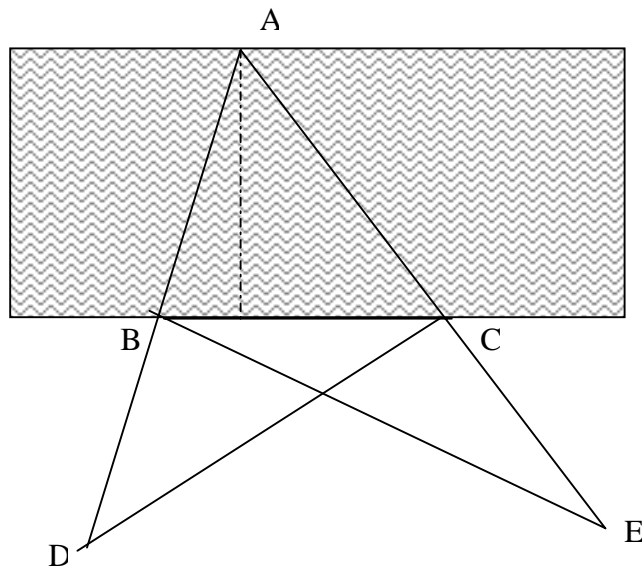
NAME _____ DATE _____

RIVER WIDTH

A surveyor has been hired to find the width of the Illinois River. Survey points are located as follows: A is on one side of the river; B and C are on the other side; D is in line with AB, and E is in line with AC as shown in the figure.

\overline{BC} measures 506.23 feet; \overline{BD} measures 453.13 feet; \overline{BE} measures 809.92 feet; \overline{CD} measures 753.61 feet, and CE measures 392.77 feet.

Find the width of the river (from A to BC) to the nearest hundredth of a foot. Explain in writing what you did and why you did each step.



Adapted from Trig Star Contest problem, 98-2, National Society of Professional Surveyors, 1998.

$$506.23^2 = \sqrt{392.77^2 + 809.92^2 - 2 \cdot 392.77 \cdot 809.92 \cdot \cos \gamma}$$

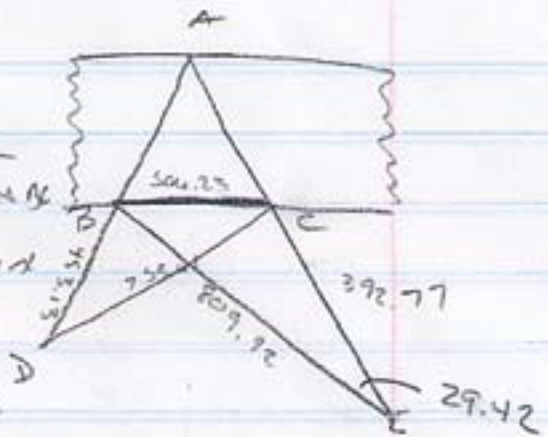
$$256268.81 = \sqrt{154268.27 + 655970.41 - 636224.56 \cos \gamma}$$

$$256268.81 =$$

$$-553969.87 = -636224.56 \cos \gamma$$

$$\cdot 871 = \cos \gamma$$

$$\gamma = 29.42$$

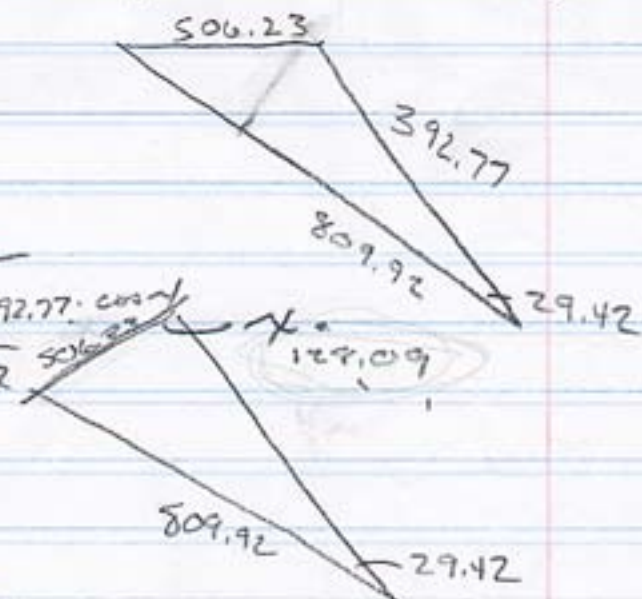


$$809.92^2 = \sqrt{506.23^2 + 392.77^2 - 2 \cdot 506.23 \cdot 392.77 \cdot \cos \gamma}$$

$$655970.41 = \sqrt{410557.09 - 397668.942 \cos \gamma}$$

$$\cos \gamma = -0.617$$

$$\gamma = 128.09$$

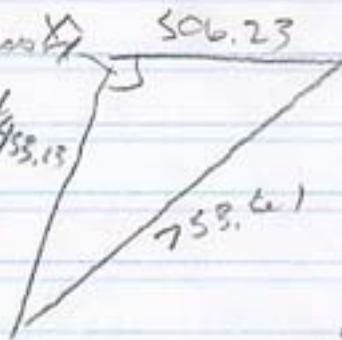


$$753.61^2 = \sqrt{461595.60 - 458775.99 \cos \gamma}$$

$$106332.42 = 458775.99 \cos \gamma$$

$$\cos \gamma = -0.232$$

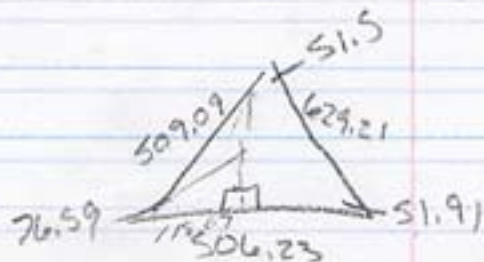
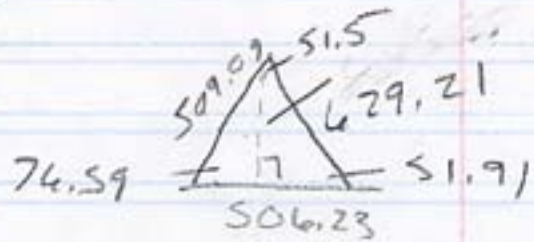
$$\gamma = 103.41$$



$$\frac{\sin 51.5}{506.23} = \frac{\sin 51.91}{\gamma}$$

$$398.42 = \sin 51.5 \gamma$$

495, 22



Find the angle of $\angle AEB$ using cosine law

Then use it to find angle $\angle BCE$

Now find angle $\angle BDC$ using cosine

Now you have 2 \angle s for $\triangle BAC$ plus side BC

Use sine law to find side \overline{BA} & AC

Use tan to find $\angle BA$;