SAVE YOUR CHANGE

Performance Standard (6B/6C/8A).J

Analyze the use of the annuity formula and compare it to the two compound interest formulas accordingly. It is assumed students have worked with and are familiar with the annuity formula and compound interest formulas:

- *Mathematical knowledge*: know how to use the compound periodic and continuous interest formulas; know the differences and similarities between different forms of growth formulas,
- *Strategic knowledge*: use appropriate strategies to solve the problem, and
- *Explanation*: explain completely and clearly what was done and why it was done.

Procedures

1. Provide students with sufficient learning opportunities to develop the following skills in order to (6B) investigate, represent, and solve problems using number facts, operations and their properties, algorithms, and relationships, (6C) compute and estimate using mental mathematics, paper-and-pencil methods, calculators and computers, and (8A) describe numerical relationships using variables and patterns:
   - Identify, represent and solve problems with numbers expressed in exponential, logarithmic and scientific notation using technology,
   - Develop fluency in operations with real numbers, vectors, and matrices using mental computation or paper-and-pencil calculations for simple cases and technology for more-complicated cases,
   - Use the correct number of digits in computation to achieve an appropriate unit or level of accuracy when solving problems,
   - Explain the differences and similarities between different forms of growth formulas.
2. Provide each student a copy of the "Save Your Change" task sheet and the rubric. Have students review and discuss the task to be completed and how the rubric will be used to evaluate it.
3. Ask students to solve the following problem and show their work and explain their reasoning. They should be monitored by the teacher but encouraged to do their own thinking.

   Use an annual interest of 9% and the following formula to solve the problems: $S = R\dfrac{(1+i)^n - 1}{i}$ where R is the amount of the payment, i is the interest rate per payment period (as a decimal), n is the total number of payments and S is the total amount.
   (1) Use the formula and logarithms to determine how many years of home exercise would amount to $ 44,572. Use the formula again to determine how many years of investing $12 a month would amount to $ 22,134. Do your answers for #1 and #2 agree (within 1 year)? a) If so, how old does the author believe the reader to be? b) If not, how much do they differ, and what might be the reason?
   (2) Use the compound interest formula to find how much money would have to be invested today in a lump sum and left to accumulate interest compounded quarterly for 40 years at 8% interest to equal the $ 56,092 from the cappuccino example.
   (3) How much less would you have to invest if continuously compounded at 8% for 40 years to accumulate the same $ 56,092?
4. Evaluate each student's work using all 3 dimensions of the rubric and its guide to determine the performance level. Use the standard rubric, giving a score for each category for each question. Minor computational errors might include rounding errors. Use of the incorrect formulas or using the wrong interest amount per payment period should result in no more than a 2 in mathematical knowledge. A4 in mathematical knowledge would require correct answers as follows:
   - 30.9 years, 30.08 years, about 30 years old (retire at 60)
   - $2359.79
   - $73.36 ($2,359.79-$2,286.43)

   A4 in strategy would require using the correct formulas, converting annual rates to periodic rates, and months to years.

Examples of Student Work follow

Time Requirements

- 30 minutes

Resources

- Copies of the "Save Your Change" task sheet
- Calculator
- Mathematics Rubric

NAME _____ DATE _____

"SAVE YOUR CHANGE
IT COULD GROW INTO THOUSANDS OF DOLLARS"

Student Task Sheet

That was the headline for the Personal Finance column of the USA Weekend Magazine, May 29-31, 1998. The article claimed the following:

"Save more!... The first ever "National Pay Yourself Day" created by Fidelity Investments and supported by 300 corporations, was May 7. .... How to save more?

- Exercising at home instead of joining a gym saves $ 300 a year. Value at retirement: $44,572.

- Renting a video once a month instead of going to the movie (and eating popcorn) saves $12 a month. Value at retirement: $22,134.

- Having a cappuccino every other day instead of daily saves $7 a week. Value at retirement: $ 56,092."

The rate of interest used was 9% annually. The formula used was as follows:

$$S = R\,\frac{(1+i)^n - 1}{i}$$

where R is the amount of the payment, i is the interest rate per payment period (as a decimal), n is the total number of payments and S is the total amount.

Show all work and write in words what you did and why you did each step.

1. Use the formula and logarithms to determine how many years of home exercise would amount to $ 44,572.

   Use the formula again to determine how many years of investing $12 a month would amount to $ 22,134.

   Do your answers for #1 and #2 agree (within 1 year)?
   a) If so, how old does the author believe the reader to be?
   b) If not, how much do they differ, and what might be the reason?

2. Use the compound interest formula to find how much money would have to be invested today in a lump sum and left to accumulate interest compounded quarterly for 40 years at 8% interest to equal the $ 56,092 from the cappuccino example.

3. How much less would you have to invest if continuously compounded at 8% for 40 years to accumulate the same $ 56,092?

Adapted from Personal Finance, Chatzky, J., USA Weekend Magazine, May 29-31, 1998

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#1 - Part one

$$44572 = 300\frac{(1+.09)^n - 1}{.09}$$

$$\frac{44572(.09)}{300} + 1 = n\log 1.09$$

$$\log 14.37 = n\log 1.09$$

$$n = \frac{\log 14.37}{\log 1.09}$$

n = 30.93 years

* In solving this problem, I put $44,572 in for S, 300 in for R, + .09 in for i. After setting up my equation, I isolated the number w/ the exponent. I did this by taking 44,572, multipling it by .09, then divided by 300. After that I added the 1. Then to solve for the exponent, I did the log of both sides and solved for n.

#1 - Part Two

$$22134 = 12\frac{(1+.0075)^n - 1}{.0075}$$

$$\frac{22134(.0075)}{12} + 1 = 1.0075^n$$

$$14.83 = 1.0075^n$$

$$\log 14.83 = n\log 1.0075$$

$$n = \frac{\log 14.83}{\log 1.0075} = \frac{360.90}{12} = 30.08 \text{ years}$$

* In solving this problem, I put $22134 in for S, 12 in for R, + .0075 in for i instead of .09 because .09 needed to be divided by 12 since it was a monthly invest. After that, I isolated the number w/ the exponent. By doing this I multiplied 22134 by .0075 then divided by 12, and then added 1. Then I took the log of both sides and solved for n.

#1 - Part Three

a) Yes they do agree within a year. Around 60 because that is a good retirement age

#2 $A = P\left(1 + \frac{i}{c}\right)^{C \cdot N}$

$56092 = P\left(1 + \frac{.08}{4}\right)^{4 \cdot 40}$

$56092 = P(1.02)^{160}$

$\frac{56092}{(1.02)^{160}} = P$

$P = \$2,359.79$

*In this problem I set up my equation by putting 56,092 in for A, .08 in for i, 4 in for C, because it's a quarterly problem, + 40 in for N. To solve this equation, I divided .08 by 4, added 1, then took that to the 160 power. Then I took 56092 and divided it by the number I got from the other side of the equation, then solved for P.

#3. $P_t = P_o e^{i \cdot N}$

$56092 = P_o e^{.08(40)}$

$56092 = P_o e^{3.2}$

$P_o = \frac{56092}{e^{3.2}} \quad P_o = \$2286.43$

2359.79
-2286.43
$\$73.36$

*In this problem I substituted 56,092 in for A_t, .08 in for i, + 40 in for N. I then multiplied .08 + 40 to get 3.2 as the root for e. Then to solve for P_o, I took 56092 and divided it by $e^{3.2}$. This gave me $2286.43 for P_o. I then took my answer from #2, $2,359.79, and subtracted my answer from #3, $2286.43, to get my final answer, $73.36, because the problem asked how much less you would have to invest, which was $73.36 when I subtracted my two numbers.

NAME "Exceeds" (page 1) DATE

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(1) $\frac{44,572}{300} = \frac{300}{300} \cdot \frac{(1+.09)^n - 1}{.09}$

$(.09)\ 148.57 = \frac{(1.09)^n - 1}{.09} \times .09$

$13.3716 = (1.09)^n - 1$

$\frac{\log 14.3716}{\log 1.09} = \frac{n \log 1.09}{\log 1.09}$

$n = 30.93$ years

I plugged all the numbers into the formula. I divided 44,572 by 300 and then multiplied it by .09 because I wanted to get rid of the fraction. Then I added 1 to both sides to isolate the variable. I took the log of both sides to get the exponent by itself. Then I divided both sides by log 1.09 to solve for n.

$22,134 = 12 \times \frac{\left(1 + \frac{.09}{12}\right)^n - 1}{\frac{.09}{12}}$

$1844.5 = \frac{\left(1 + \frac{.09}{12}\right)^n - 1}{\frac{.09}{12}}$

$13.83375 = \left(1 + \frac{.09}{12}\right)^n - 1$

$14.83375 = \left(1 + \frac{.09}{12}\right)^n$

$\frac{\log 14.83375}{\log\left(1 + \frac{.09}{12}\right)} = \frac{n \log\left(1 + \frac{.09}{12}\right)}{\log\left(1 + \frac{.09}{12}\right)}$

$\frac{360.93}{12}$

$n = 30.08$ years

I divided .09 by 12 because it was paid monthly.

I plugged all the numbers into the formula. I divided 22,134 by 12, multiplied it by $\frac{.09}{12}$, then added 1 to both sides to get the exponent and its number by itself. I took the log of both sides and divided both sides by $\log\left(1 + \frac{.09}{12}\right)$. Then I solved for n.

a) 30 years

(2) $56,092 = P\left(1 + \frac{.08}{4}\right)^{4 \cdot 40}$

$56,092 = 23.76990696\,P$

$P = \$2,359.79$

I pluged all the numbers into the formula. Then I figured up what was in the parenthesis and took it to the 4.40 power. Then I divided 56,092 by 23.76990696 and solved for P.

(3) $56,092 = P_o\, e^{.08(40)}$

$P_o = \$2,286.43$

$A = \$73.36$

I pluged all the numbers into the formula. Then I divided 56,092 by $e^{.08(40)}$ to solve for P_o. When I got P_o I subtracted it from 2,359.79 to get how much less I paid.