

EQUAL AREA TRIANGLES

Performance Standard (7A/9A/9C).I

Determine multiple ways of dividing a generic triangle into four triangles of equal area accordingly:

- *Mathematical knowledge*: know the properties of the medians or other auxiliary lines of triangles and the information needed to calculate the area of a triangle,
- *Strategic knowledge*: use appropriate strategies to solve the problem, and
- *Explanation*: explain completely and clearly what was done and why it was done.

Procedures

1. Provide students with sufficient learning opportunities to develop the following skills in order to (7A) measure and compare quantities using appropriate units, instruments and methods, (9A) demonstrate and apply geometric concepts involving points, lines, planes and space, and (9C) construct convincing arguments and proofs to solve problems:
 - Calculate by an appropriate method the length, width, height, perimeter, area, volume, surface area, angle measures or sums of angle measures of common geometric figures, or combinations of common geometric figures,
 - Describe and apply properties of polygons and circles in a problem-solving situation,
 - Solve problems using constructions,
 - Identify and apply properties of medians, altitudes, angle bisectors, perpendicular bisectors and midlines in triangles,
 - Justify constructions using geometric properties, and
 - Develop conjectures about geometric situations with and without technology.
2. Provide each student a copy of the "Equal Area Triangles" task sheet and the rubric. Have students review and discuss the task to be completed and how the rubric will be used to evaluate it.
3. Ask students to complete the following task in a classroom setting:

Divide triangle ABC into 4 triangles with equal areas, based on geometric principles. Sketch two other ways to divide triangle ABC into four triangles with equal areas and explain in words what you did and why you did each step. Justify your solution using geometric properties.

4. Evaluate each student's work using all 3 dimensions of the rubric and its guide to determine the performance level. For a score of 4 in mathematical knowledge, the student should complete two correct solutions of the more than 30 possible solutions. For a 3 in mathematical knowledge, the student must have one correct solution and a nearly correct second solution. For a 4 in strategy, the student must utilize medians or other auxiliary lines to divide triangles into equal area triangles. For a 4 in explanation, the student must be precise in the description of the use of medians and why they will guarantee equal areas.

Examples of Student Work follow

Time Requirements

- One class period

Resources

- Copies of the "Equal Area Triangles" task sheet
- Straight edges or rulers
- Compasses (optional)
- Mathematics Rubric

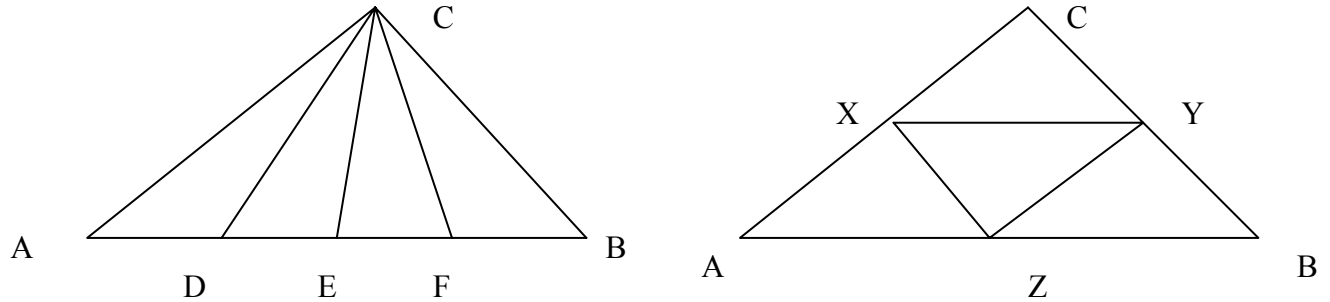
ASSESSMENT (7A/9A/9C).I

NAME _____ DATE _____

EQUAL AREA TRIANGLES

Student Task Sheet

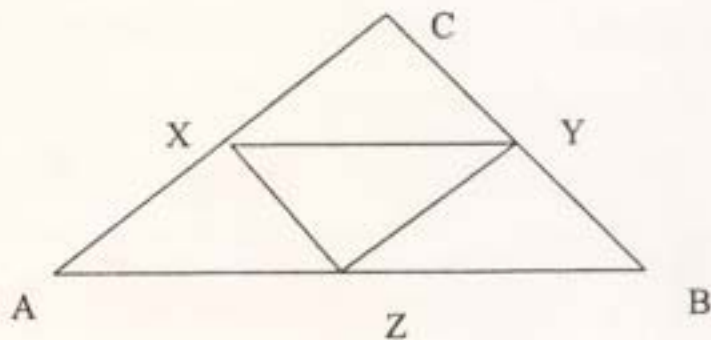
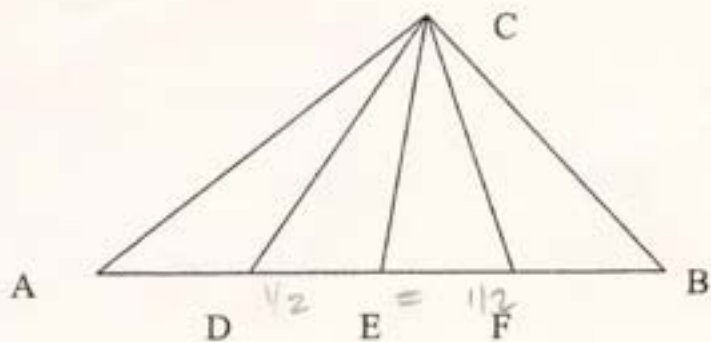
Divide triangle ABC into 4 triangles with equal areas, based on geometric principles. One example is shown below left, where E is the midpoint of AB, D is the midpoint of AE, and F is the midpoint of EB. The other example below right uses midlines.



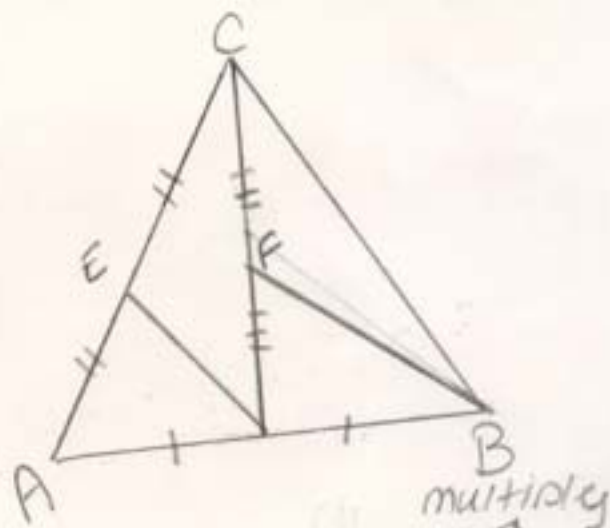
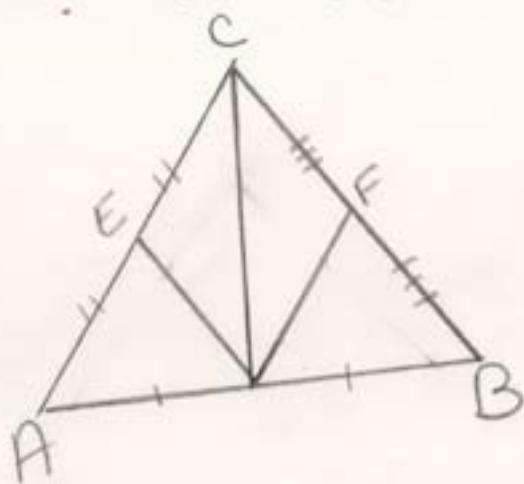
Sketch two other ways to divide triangle ABC into four triangles with equal areas and explain in words what you did and why you did each step. Justify your solution using geometric properties. Describe in words what geometric properties you are using and why the areas must be the same.

Adapted from "A Triangle Divided", Mathematics Teacher, October 2000, pp. 608-611.

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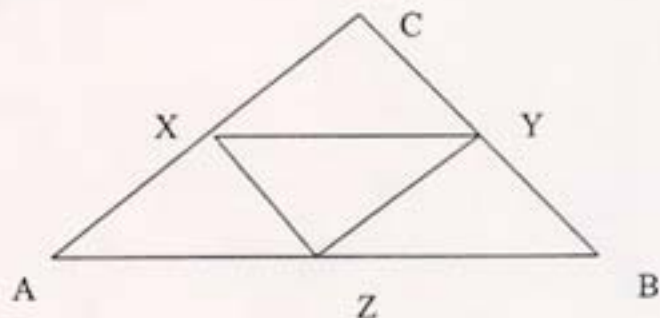
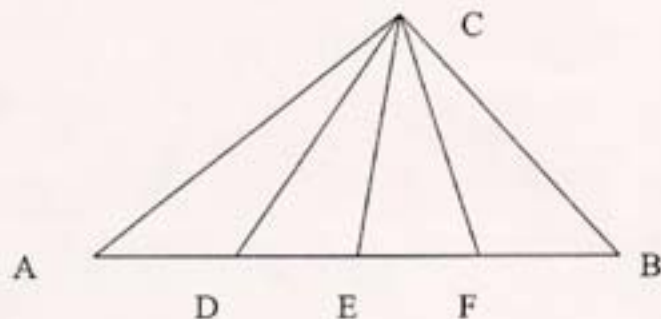
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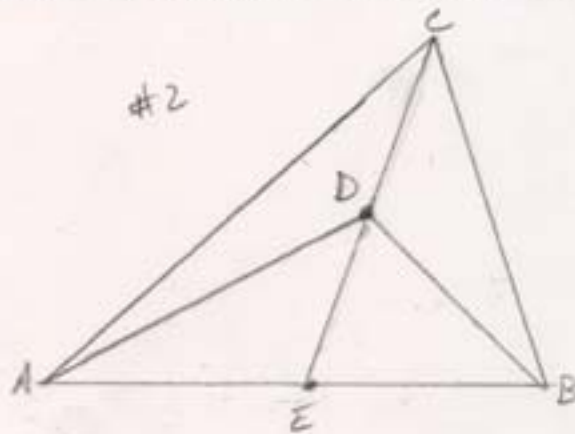
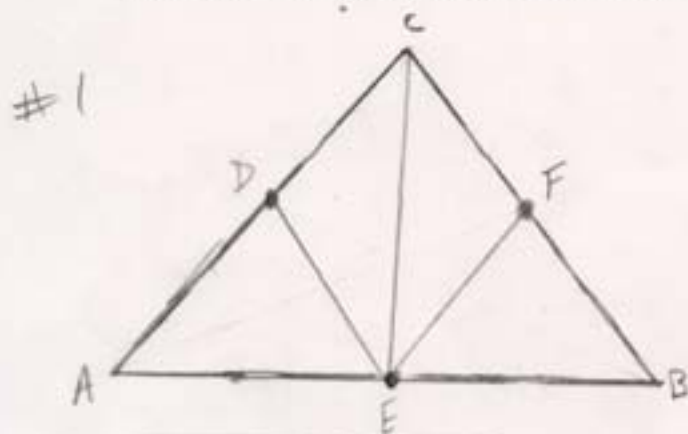
The area of a triangle is $\frac{1}{2} \times \text{base} \times \text{height}$.
 if you take $\frac{1}{2}$ and divide it into 2, then the area will be the same. then divide the 2 halves again to get equal parts.

Student Task Sheet

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#1
 E is the midpoint of AB
 D is the midpoint of AC
 F is the midpoint of CB
 If CE is the median of $\triangle ABC$,
 then $\triangle ACE$ & $\triangle ECB$ has equal area because of same base & same altitude.
 If DE is the median of $\triangle AEC$ on
 $\triangle AEC$, then $\triangle AED$ & $\triangle DEC$ have equal area because of same base & altitude.
 If EF is the median of $\triangle ECB$, then $\triangle EFC$ & $\triangle EFB$ have equal area because
 of same base & altitude.
 So all areas are equal

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2
CE is the median of $\triangle ABC$. $\triangle AEC$ and $\triangle EBC$ have equal areas because of same base & altitude.

If AD is the median of $\triangle AEC$, then $\triangle ADC$ and $\triangle ADE$ have equal area because of same base & height.

If DB is the median of $\triangle BCE$, then $\triangle BDC$ and $\triangle BDE$ have equal areas because of same base & height. Therefore all four \triangle 's have an equal area.