DIAGONALS AND ALTITUDES

Performance Standard 9D.H

Find the length of the diagonal of a square that has a side the length of the altitude of an equilateral triangle, explain reasoning and justify procedures.

- *Mathematical knowledge*: Demonstrate the side length relationships in 45-45-90 triangles and 30-60-90 triangles;
- *Strategic knowledge*: Solve problem using complete systematic process;
- *Explanation*: Explain completely what was done and why it was done.

Procedures

1. Provide students with sufficient learning opportunities to develop the following in order to use trigonometric ratios and circular functions to solve problems.
   - Determine and justify the side length relationships present in 45-45-90 triangles and 30-60-90 triangles.
2. Provide students with the assessment task worksheet. Have students work individually. Calculators may be used.

   The picture below shows an equilateral triangle and a square of equal height. You are given that the triangle has a side length of 2 units. Using your knowledge of the special right triangles formed in these figures, determine the length of the diagonal of the square. Make sure to explain your reasoning and justify your procedures.
3. Use the standard scoring rubric. Give each student a score in each of the three categories. A score of 4 should indicate completely correct solutions to all parts of the problem, with complete and correct justifications of their reasoning. A three should represent correct or nearly correct solutions to all parts, with only minor computational errors making their solutions inaccurate, there rationale should be sound, but may not be completely explained. A two would indicate that students have some idea about how to answer the questions, but make major errors in computation and or reasoning that effects their answers. A one may have a correct answer for one part, but generally shows little understanding in their rationale for their procedures and processes. A score of zero generally reflects no correct responses and no logical rationale for their procedures and processes.
4. Minor errors in computation include making errors in the actual addition or multiplication, rounding incorrectly. Major errors include using the wrong operations or formulas to relate terms.
5. The students who receive a four on this task, should realize that the altitude of an equilateral triangle is the longer leg of a 30-60-90 triangle, and that length is $\sqrt{3}$ times the length of the shorter leg. The shorter leg is half the hypotenuse, which is the length of the side of the equilateral triangle. That means that the shorter leg is 1, and the altitude is $\sqrt{3}$. Since this is the same as the length of the side of the square, and the diagonal of the square forms 45-45-90 triangles, the diagonal must be $\sqrt{2}$ times the length of the side, which ends up being $\sqrt{6}$. Students may arrive at this same conclusion using Pythagorean theorem and not using the special properties of these right triangles, however this rationale would not demonstrate this standard. If they must revert to the Pythagorean theorem to solve this problem, but do include discussion of the special right triangle relationships they can still earn a 3 on mathematical knowledge. If they do not even mention the special right triangle relationships they should receive no more than a 2 for mathematical knowledge on this task.

Examples of Student Work follow

Time Requirements

- One class period

Resources

- Copies of the “Diagonals and Altitudes” task sheet
- Writing utensil
- Calculators may be used
- Mathematics Rubric

NAME ______________________________ DATE ____________________

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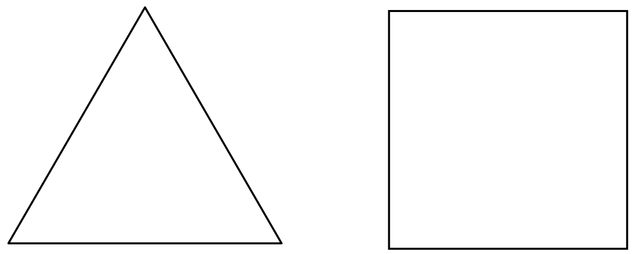
Student Task Sheet

The picture below shows an equilateral triangle and a square of equal height. You are given that the triangle has a side length of 2 units. Using your knowledge of the special right triangles formed in these figures, determine the length of the diagonal of the square. Make sure to explain your reasoning and justify your procedures.



"Meets"

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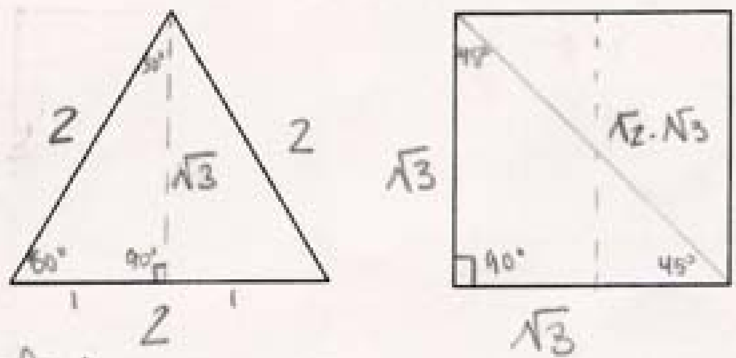

! Important Info !

- triangle = 2x2x2

If you divide an equilateral triangle in two halfs, you would get 2 triangles who's angles are 30-60-90 the theory is that if leg A equals x, then the hypotenuse would be 2x. If you divide one side by two, you would get 1. The other side would still stay 2 so it fits the theory. The leg B of this triangle would be $x\sqrt{3}$. If $x=1$, the the leg B equals $\sqrt{3}$. So if the figures share the same height, the the square's side would be $\sqrt{3}$. If you divide a square in to two pieces, you would get two triangles who's measures are 45-45-90. The theory says that if you have a 45-45-90 triangle then legs A & B will equal x, which in this case is $\sqrt{3}$. The hypotenuse will equal $x\sqrt{2}$. So in this case it would be $\sqrt{3} \cdot \sqrt{2} = \sqrt{6}$. So the diagonal of the square & the hypotenuse of the 45-45-90 triangle equals $\sqrt{6}$.

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Student Task Sheet

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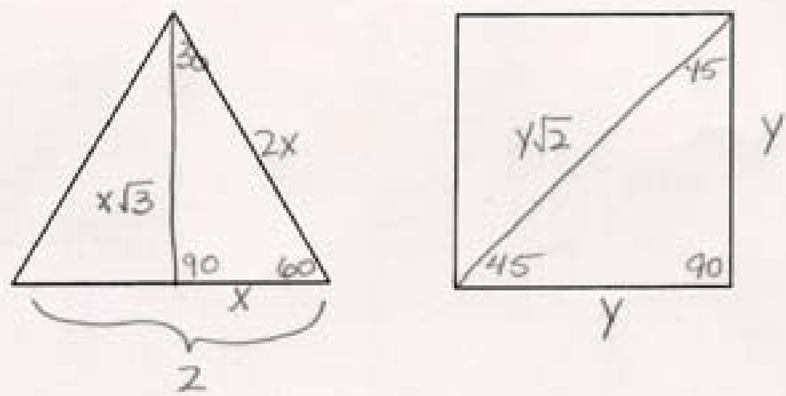

- $x=1$ height of triangle $=\sqrt{3}$ hypotenuse $=2$
- $x\sqrt{3}=y$, $y=\sqrt{3}$, $(\sqrt{3})(\sqrt{2})=\sqrt{6}$

The length of the diagonal of the square is $\sqrt{6}$. I used the strategy of finding out the height of the equilateral triangle. Since the heights of both figures are equal, the height of the triangle is also the height of the square, Since the length of the height of the square is equal to the length of one of its sides, knowing the height of the square enables me to find the diagonal of the square. The first thing I did was draw the altitude of the equilateral triangle. Since the definition of an altitude is the line segment bisecting the vertex of a triangle perpendicular to the opposite side of the vertex, I knew that the altitude would create a right triangle. I also knew that a equilateral triangle is an equiangular triangle, which means each of its angles are 60°. Since an altitude bisects one vertex of the triangle, one of the smaller right triangle's angles is 30°. The remaining angle is 60° because it is a regular angle of an equiangular triangle The smaller triangles created when I drew the altitude has 3 angles, measuring 30°, 60°, and 90°. That would make it a special 30°-60°-90° right triangle. I wanted to make the altitude because I could then use the formulas that applied to the special 30-60-90 right angles. In a 30-60-90 triangle all the lengths can be found out by knowing the shortest leg's length. I labeled that length x on the picture The hypotenuse's length (of the 30-60-90 triangle) is $2x$ and the longer leg is $x\sqrt{3}$. The hypotenuse of the 30-60-90 triangle is 2 (given) because the hypotenuse of the 30-60-90 triangle is also a side of the equilateral triangle. I used that information and the fact that the length of the hypotenuse is twice the length of the shortest leg to find out what the length of the shortest leg is. I did this because I needed to find out the length of the shorter leg to find the length of the longer leg (which is the height of the triangle). I used the equation $2x$ (length of hypotenuse relative to length of shorter leg) $= 2$ (actual length of hypotenuse). After solving the equation, I got $x=1$, which meant the length of the shorter leg was 1. Using the fact that the length of the longer leg is $x\sqrt{3}$ (x being length of the shorter leg), I replaced x with 1 and evaluated the expression, $1\sqrt{3}$. The answer of $1\sqrt{3}$ is $\sqrt{3}$, so the length of the longer leg of the 30-60-90 triangle is $\sqrt{3}$. As I said before, the length of the longer leg of the 30-60-90 triangle is also the height of the equilateral triangle. The height of the equilateral triangle is also the height of the square Therefore, both the height of the triangle and square is $\sqrt{3}$. The height of the square is also the length of one of its sides, so the side's length is $\sqrt{3}$. From that point, I drew the square's diagonal, creating an isosceles right triangle. I created the isosceles right triangle because then I could use the formulas to find the hypotenuse of the isosceles right triangle. The hypotenuse of the isosceles right triangle is also the diagonal of the square, so if I find the hypotenuse, I also find the length of the diagonal. The formula for Finding the hypotenuse of an isosceles triangle relative to one of its legs is $y\sqrt{2}$, y being the length of one of its legs. Since the length of one of the isosceles triangle's legs is $\sqrt{3}$, I substituted y for $\sqrt{3}$ in $y\sqrt{2}$ to find the length of the hypotenuse of the isosceles right triangle $(\sqrt{3})(\sqrt{2}) = \sqrt{6}$, so the length of the hypotenuse is $\sqrt{6}$. The length of the hypotenuse of the isosceles right triangle is the same as the length of the diagonal of the square, so the length of the diagonal of the square is $\sqrt{6}$.