

MARCY'S DOTS

Performance Standard 8A.H

Identify and extend the pattern of changes in a series of pictures of dots and generalize that pattern in words, and symbols either recursively or explicitly accordingly:

- *Mathematical knowledge:* Identify linear, quadratic and exponential patterns and how to write algebraic expressions to describe a sequence.
- *Strategic knowledge:* Investigate linear, quadratic, and exponential patterns recursively.
- *Explanation:* Explain completely and clearly what was done and why it was done.

Procedures

1. ***In order to describe numerical relationships using variables and patterns (8A)***, students should experience sufficient learning opportunities to develop the following:
 - Investigate and describe linear, quadratic, and exponential patterns recursively.
 - Investigate and write algebraic expressions to describe the n th term of a simple linear, power or exponential sequence.
2. Provide each student a copy of the “Marcy’s Dots” task sheets and the rubric. Have students review and discuss the task and how the rubric will be used to evaluate it.
3. Have students solve the problem on the task sheet. (Do not provide help or guidance to the students.)
4. Evaluate each student’s work using the rubric and its guide to determine the performance level. Give each student a score in each of the three categories. Minor computation errors include any error in counting or multiplication, as well as errors in copying from one place to another. Major errors include using the wrong operation. Students should describe the process of adding one row of dots and an extra dot to each row to get from one step to the next. Other descriptions are also possible, such as adding 5, then 7, then 9, then 11 dots, etc. In other words the number of dots added between steps increases by two each time.
 - A correct response for the number of dots in the 20th step is 440. There are several ways of getting this. One way of getting this is to realize that the 19th step had 399 dots in 19 rows; then there were 21 dots per row. Adding one more row to get 20 rows and an extra dot per row to get 22 dots per row, would give a total of 440 dots. Another method may be to say it was increasing from the 19th step to the 20th step, and looking at the pattern in the numbers of dots it increased, the 19th increase would be the first increase plus two additional dots for each increase after that one, which is $5 + 2(18)$ or 41 dots. (This can also be seen as $2n+1$ dots added for step n .) $399 + 41$ is 440.
 - The 30th step should have 960 dots. A mathematical knowledge score of a 4 should include a generalization of the pattern in terms of an algebraic expression or a written description. If an algebraic expression is attempted for the pattern, it is $n(n+2)$ or the equivalent, and a written description would describe the dot pattern as a rectangle that is two more dots wide than it is high, and include the fact that the number of dots high is the same as the step number. (If they attempt to write a formula recursively in a generalized form, they need to subscript variables and write something like: $a_n = a_{n-1} + (5 + 2(n-2))$ OR $a_1 = 3$ and $a_n = a_{n-1} + (2n + 1)$. Attempts to write an algebraic expression for a recursive pattern without subscripts is not acceptable. Students need to realize that n cannot represent both the value of the term and the step number.)
 - If the student solves the problem correctly by drawing all 20 or 30 pictures, the strategy score should be no more than a 2, since they did not follow the directions.
 - Part D should not require students to solve a quadratic. The idea is that if they know that the 30th step has 960 dots, there is no way to get to 1000 dots by following the pattern of increases.The explanations should include how they found these answers, as well as why these answers are correct.

Examples of Student Work follow

Resources

- Copies of the “Marcy’s Dots” task sheet
- Paper and pencil
- Calculators (optional)
- Mathematics Rubric

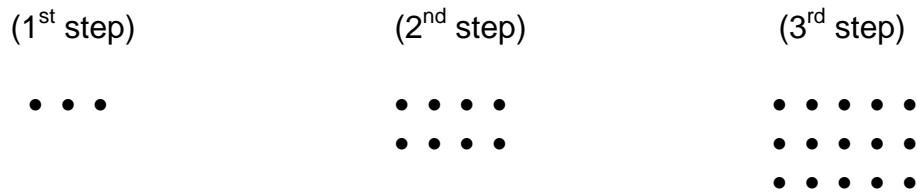
Time Requirements

- One class period

MARCY'S DOT PROBLEM

This question requires you to show your work and explain your reasoning. You may use drawings and numbers in your explanations, but you must provide a written description to go with these, that explains what you did and why you did it. Your answer should be clear enough that another person could read it and understand your thinking. It is important to show all your work.

A pattern of dots is shown below. At each step, more dots are added to the pattern. The number of dots added at each step is enough to allow the pattern of dots to continue to grow in the manner shown. The pattern continues infinitely.



- A. Describe the change that takes place to get from one step to the next. Make your description in general terms, so that it is good for each succeeding step in the pattern.
- B. Marcy has to determine the number of dots in the 20th step, but she does not want to draw all 20 pictures and then count the dots. Her friend told her that there were 399 dots in the 19th step. How can she use this to find the number of dots in the 20th step?
- C. Marcy's teacher knows that her friend helped her. So she asks her to explain or show how she could find the number of dots in the 30th step without drawing all 30 pictures. Explain how Marcy can do this AND give the answer that Marcy should get for the number of dots in the 30th step.
- D. Does any step in this pattern contain exactly 1000 dots? Explain your reasoning.

Marcy's Dot Problem

This question requires you to show your work and explain your reasoning. You may use drawings and numbers in your explanations, but you must provide a written description to go with these, that explains what you did and why you did it. Your answer should be clear enough that another person could read it and understand your thinking. It is important to show all your work.

A pattern of dots is shown below. At each step, more dots are added to the pattern. The number of dots added at each step is enough to allow the pattern of dots to continue to grow in the manner shown. The pattern continues infinitely.

(1st step)

• • •

(2nd step)

• • • •
• • • •

(3rd step)

• • • • •
• • • • •
• • • • •

- A. Describe the change that takes place to get from one step to the next. Make your description in general terms, so that it is good for each succeeding step in the pattern.

The dots in the lines just keep increasing by 1 in each step.

- B. Marcy has to determine the number of dots in the 20th step, but she does not want to draw all 20 pictures and then count the dots. Her friend told her that there were 399 dots in the 19th step. How can she use this to find the number of dots in the 20th step?

She should divide 399 by 19, and get 21. That is the # of dots in one line of the 19th step. Then she just needs to add 1 more dot in each line for the 20th step. That would get her 22 dots in a line. Then she just multiplies 22 by 20 & the answer would be 440.

- C. Marcy's teacher knows that her friend helped her. So she asks her to explain or show how she could find the number of dots in the 30th step without drawing all 30 pictures. Explain how Marcy can do this AND give the answer that Marcy should get for the number of dots in the 30th step. She has to notice the pattern,

which is as follows. 1st step has 3 dots, 2nd step has 4 dots, 3rd step has 5 dots, ... Each time step has 2 more dots than its self #. So she has to add 2 on 30 which is 32 & multiply that by 30 which is 960 that's her answer.

- D. Does any step in this pattern contain exactly 1000 dots? Explain your reasoning.

No because step 30 has 960 & if you figure out how many step 31 has which is 1023 there is no way you can get a 1,000.

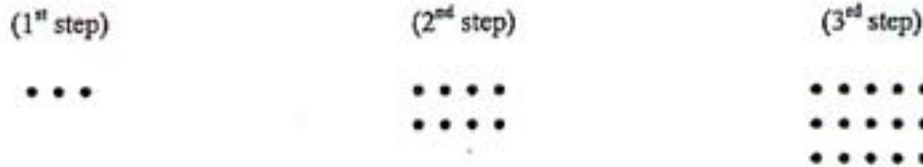
Name Alex

Date 4-5-01

Marcy's Dot Problem

This question requires you to show your work and explain your reasoning. You may use drawings and numbers in your explanations, but you must provide a written description to go with these, that explains what you did and why you did it. Your answer should be clear enough that another person could read it and understand your thinking. It is important to show all your work.

A pattern of dots is shown below. At each step, more dots are added to the pattern. The number of dots added at each step is enough to allow the pattern of dots to continue to grow in the manner shown. The pattern continues infinitely.



- A. Describe the change that takes place to get from one step to the next. Make your description in general terms, so that it is good for each succeeding step in the pattern.

The 1st thing you do is add a dot to the end of the row. Then you add another row with the same amount of dots.

- B. Marcy has to determine the number of dots in the 20th step, but she does not want to draw all 20 pictures and then count the dots. Her friend told her that there were 399 dots in the 19th step. How can she use this to find the number of dots in the 20th step?

In the 19th step there will be 19 rows. So you divide 399 by 19 to find out how many dots are in 1 row. There are 21 dots in each row. Then you add 1 dot to the end of each row. So now there is 22 dots. Then you add a row of 22. So now there is 20 rows of 22 dots. So take 20 x 22 and you get 440 dots.

- C. Marcy's teacher knows that her friend helped her. So she asks her to explain or show how she could find the number of dots in the 30th step without drawing all 30 pictures. Explain how Marcy can do this AND give the answer that Marcy should get for the number of dots in the 30th step.

You know that there are 440 dots in the 20th row. So to get to the 30th row 10 dots and 10 rows will be added. So just do the 10 between 20th row and the 20th row.

20 th	21 st	22 nd	23 rd	24 th	25 th	26 th	27 th	28 th	29 th	30 th
440	483	528	575	624	675	728	783	840	899	960

- D. Does any step in this pattern contain exactly 1000 dots? Explain your reasoning.

No, because the 30th step contains 960 dots. So that means each row has 32 dots. Then if we add a dot to each row and add a row then we would have 33 dots and 31 rows. Then you take the # of dots x's the # of rows and you would get 1023.

Adapted from the NAEP Trial State Assessment (1992).

19) $\frac{399}{19} = 21$
 $\frac{21 \times 20}{20} = 420$
 $420 + 20 = 440$

Just take the # of dots x the # of rows