

AROUND THE STOP SIGN

Performance Standard (7C/9C).H

Analyze a drawing of a regular octagon inscribed in a circle to determine angle measures, using knowledge of properties of regular polygons and the sums of angles in various polygons to help solve the problem.

- *Mathematical knowledge:* Use properties of regular polygons, sum of angles, parallel lines with alternate interior angles and vertical angles;
- *Strategic knowledge:* Determine angle measurements using systematic method;
- *Explanation:* Explain completely what was done and why it was done.

Procedures

1. Provide students with sufficient learning opportunities to develop the following in order to (7C) s select and use appropriate technology, instruments, and formulas to solve problems, interpret results, and communicate findings, and (9C) construct convincing arguments and proofs to solve problems.
 - Solve problems involving angle measurement in polygons and circles.
 - Create and critique arguments concerning geometric ideas and relationships, such as congruence, similarity, the Pythagorean relationship, or formulas for surface area or volume of simple three dimensional objects.
 - Represent, solve and explain numerical and algebraic relationships using geometric concepts.
2. Provide students with the assessment task worksheet. Have students work individually. As students work on the task, remind them to justify their work and calculations. Calculators may be used.

Two diameters of the circle have been drawn, along with another diagonal of the octagon. The drawing results in 26 non-overlapping angles being formed, as shown. Use the properties of circles and polygons to determine the exact measure of each of these angles. Provide a justification for each of your measures, making sure to completely explain your reasoning.
3. Use the standard scoring rubric. Give each student a score in each of the three categories. A score of 4 should indicate completely correct solutions to all parts of the problem, with complete and correct justifications of their reasoning. A three should represent correct or nearly correct solutions to all parts, with only minor computational errors making their solutions inaccurate, their rationale should be sound, but may not be completely explained. A two would indicate that students have some idea about how to answer the questions, but make major errors in computation and or reasoning that effects their answers. A one may have a correct answer for one part, but generally shows little understanding in their rationale for their procedures and processes. A score of zero generally reflects no correct responses and no logical rationale for their procedures and processes.
4. Minor errors in computation include making errors in the actual addition or multiplication, rounding incorrectly. Major errors include using the wrong operations or formulas to relate terms.
5. With the angle numbering shown on the diagram the measures should be:
Angles 1,2,3,4,8,9,10,11,20,22,23 & 25 have a measure of 67.5°
Angles 5,14,16 & 18 measure 135°
Angles 6 & 13 are 90°
Angles 7,12, 15, 17, measure 45°
Angles 19,21, 24 & 26 measure 112.5°
Rationale for these measurements should include using the idea of a regular octagon having all the same sized angles, and the sum of all 8 totaling 1080. Use of parallel lines, with alternate interior angles, and vertical angles may also be appropriate. Breaking the shape down into isosceles triangles and using their properties is also possible. You must judge to reasonableness and logic of their argument, and not accept reasoning based strictly on appearance (such as “it looks like it’s a right angle, so its 90 degrees). However, keep in mind that their justifications are not formal proofs, and thus their explanations may lack exact statements and reasoning expected in a proof. A student who earns a four on this task should successfully identify the measure of all 26 angles and justify those measures appropriately. A score of three should be reserved for students who make minor computational errors in their calculations but whose procedures and rationale are mathematically sound. Students with only some of the correct solutions and with rationales that are weak or not clear should be viewed as approaching, but not meeting standards.

Examples of Student Work follow

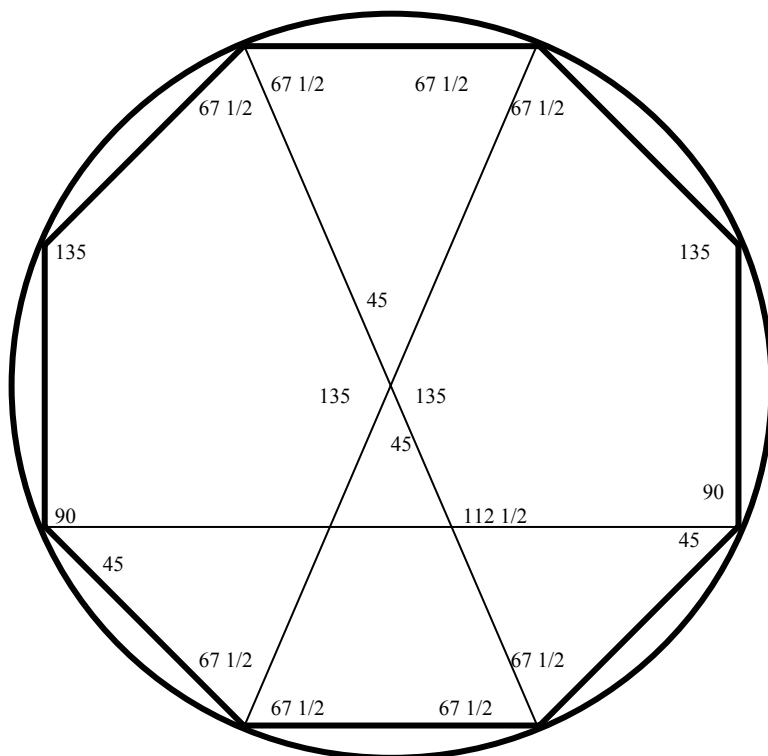
Time Requirements

- One class period
- It will not take them that long to calculate the angle measures, but they need time to write proper justifications for their calculations.

Resources

- Copies of the “Around the Stop Sign” task sheet
- Writing utensil
- Calculators may be used
- Mathematics Rubric

Answer Key:

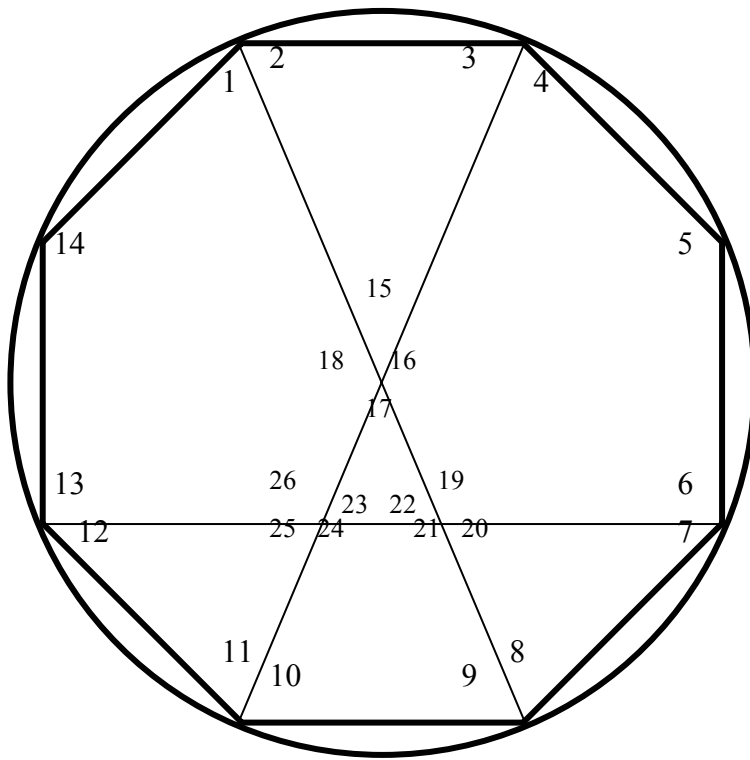


NAME _____ DATE _____

AROUND THE STOP SIGN

Student Task Sheet

The picture below shows a regular octagon inscribed in a circle. Two diameters of the circle have been drawn, along with another diagonal of the octagon. The drawing results in 26 non-overlapping angles being formed, as shown. Use the properties of circles and polygons to determine the exact measure of each of these angles. Provide a justification for each of your measures, making sure to completely explain your reasoning. (No protractor is necessary.)

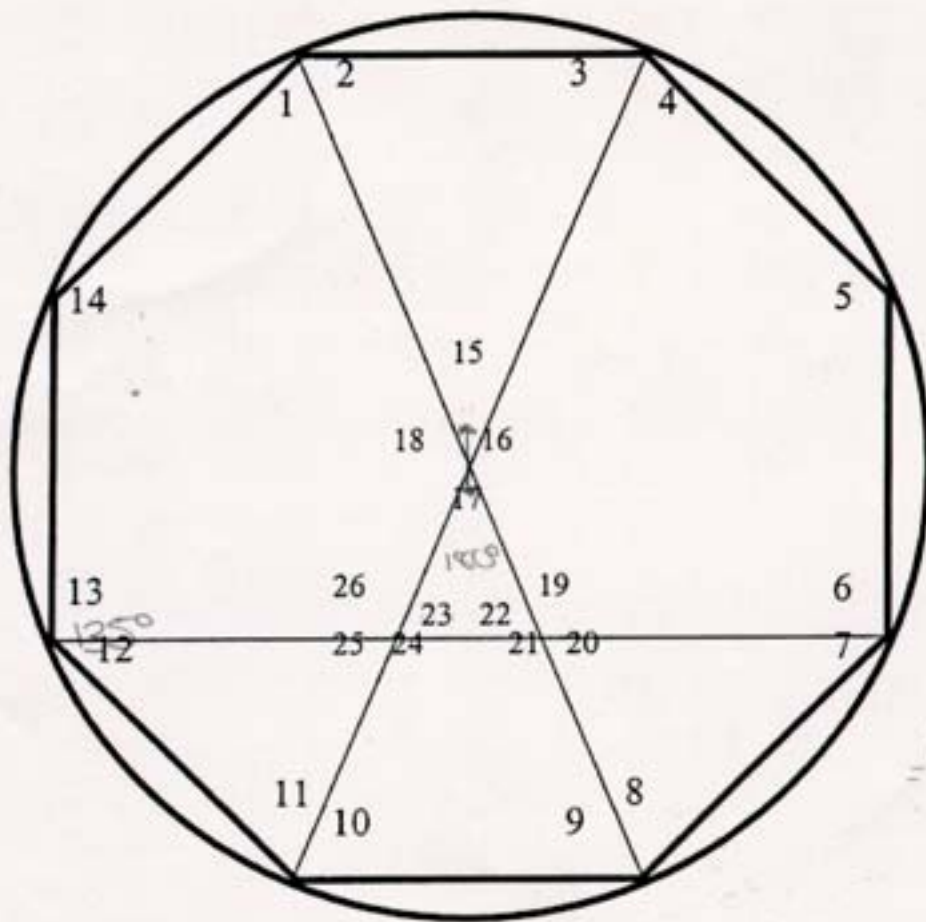


AROUND THE STOP SIGN

"Meets" (page 1)

Student Task Sheet

The picture below shows a regular octagon inscribed in a circle. Two diameters of the circle have been drawn, along with another diagonal of the octagon. The drawing results in 26 non-overlapping angles being formed, as shown. Use the properties of circles and polygons to determine the exact measure of each of these angles. Provide a justification for each of your measures, making sure to completely explain your reasoning. (No protractor is necessary.)



$$\begin{array}{r} 90 \\ + 45 \\ \hline 135^\circ \end{array}$$

$$m\angle 12 = m\angle 7 = 45^\circ$$

$$m\angle 1 = m\angle 2 = m\angle 3 = m\angle 4 = m\angle 8 = m\angle 9 = m\angle 10 = m\angle 11 = 67.5^\circ$$

$$m\angle 17 + m\angle 23 + m\angle 22 = 180^\circ$$

$$\begin{array}{l} m\angle 17 = 45^\circ \\ m\angle 23 = 67.5^\circ \\ m\angle 22 = 67.5^\circ \end{array}$$

$$\begin{array}{l} m\angle 17 = m\angle 15 \\ m\angle 15 = 45^\circ \end{array}$$

$$\begin{array}{l} m\angle 18 = m\angle 16 \\ 180^\circ - 45^\circ \\ 135^\circ \end{array}$$

$$m\angle 25 = m\angle 20 = 67.5^\circ$$

$$\begin{array}{l} m\angle 26 = m\angle 19 = m\angle 24 = m\angle 21 \\ 180 - 67.5 \\ 112.5^\circ \end{array}$$

$$m\angle 13 = m\angle 6 = 90^\circ$$

$$m\angle 14 = m\angle 5 = 135^\circ$$

The first thing I did was divided angles 1 and 2 into 90° segments.

The degree of the angle was 135° therefore both angles 1 and 2 equal 67.5° because I divided 135° by two. Since angles 8, 9, 10, 11 and 3, 4 are equal they too are 67.5° .

$m\angle 1 = m\angle 2 = m\angle 3 = m\angle 4 = m\angle 8 = m\angle 9 = m\angle 10 = m\angle 11$

These angles all equal 67.5° .

$m\angle 5 = m\angle 14 = m\angle 16 = m\angle 18$

These angles equal 135° because the angles of the octagon equal 135° and these are all sides. The diagonals are the same angles as the sides.

$m\angle 6 = m\angle 13$

These angles equal 90° because they're right angles.

$m\angle 7 = m\angle 12 = m\angle 15 = m\angle 17$

These angles equal 45° because when added to 90° they equal 135° .

$m\angle 20 = m\angle 22 = m\angle 23 = m\angle 25$

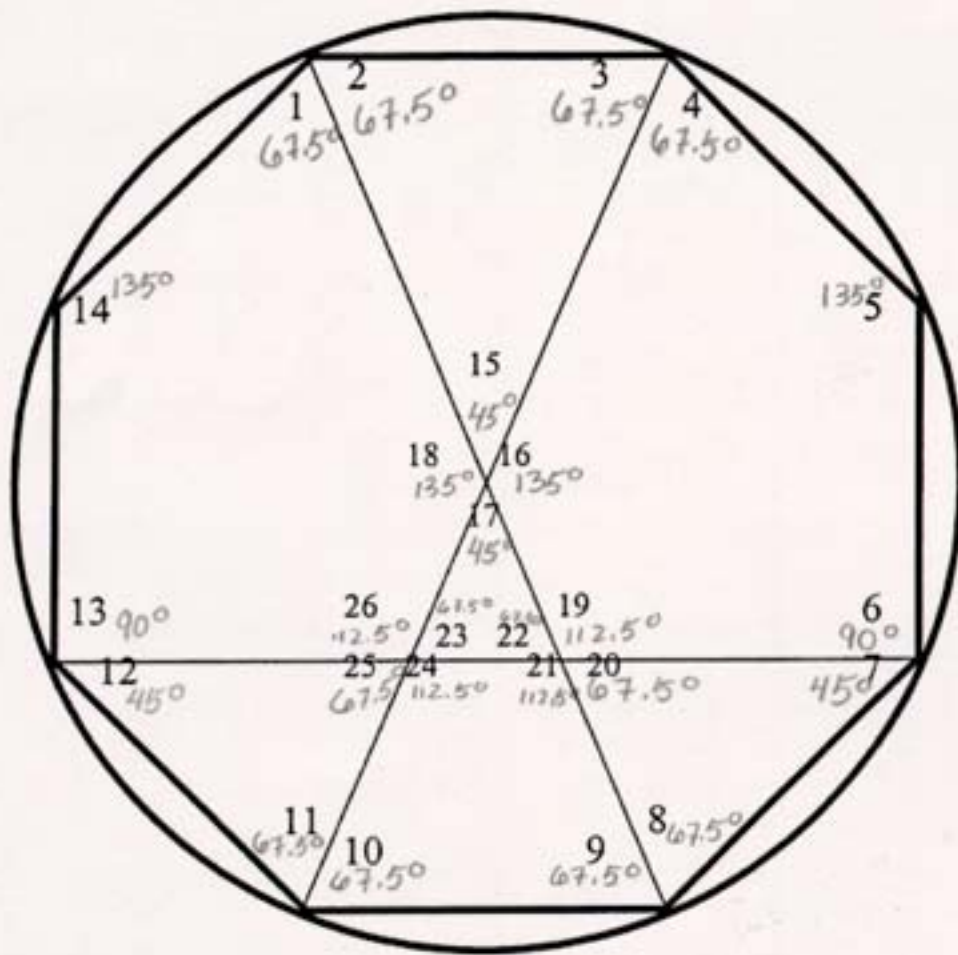
These angles equal 67.5° because they are alternate interior angles and $67.5^\circ \times 2 + 45^\circ = 180^\circ$ which is what the angles of a triangle should add up to.

$m\angle 19 = m\angle 21 = m\angle 24 = m\angle 26$

These angles equal 112.5° . They are on the same line as a 67.5° angle and a line equals 180° . Therefore $180^\circ - 67.5^\circ = 112.5^\circ$

Student Task Sheet

The picture below shows a regular octagon inscribed in a circle. Two diameters of the circle have been drawn, along with another diagonal of the octagon. The drawing results in 26 non-overlapping angles being formed, as shown. Use the properties of circles and polygons to determine the exact measure of each of these angles. Provide a justification for each of your measures, making sure to completely explain your reasoning. (No protractor is necessary.)



$$\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4 \cong \angle 8 \cong \angle 9 \cong \angle 10 \cong \angle 11 \cong \angle 20 \cong \angle 22 \cong \angle 23 \cong \angle 25 = 67.5^\circ$$

$$\angle 7 \cong \angle 12 \cong \angle 15 \cong \angle 17 = 45^\circ$$

$$\angle 19 \cong \angle 21 \cong \angle 24 \cong \angle 26 = 112.5^\circ$$

$$\angle 5 \cong \angle 14 \cong \angle 16 \cong \angle 18 = 135^\circ$$

$$\angle 6 \cong \angle 13 = 90^\circ$$

Since I already knew what degree measure each of the regular octagon's angles are (135°), then I divided angles that are formed by $\angle 1, \angle 2, \angle 3, \angle 4, \angle 8, \angle 9$, and $\angle 10, \angle 11$ by two, because the diameters of the circles bisected four of the angles of the regular octagon. Since $\frac{135}{2}$ is 67.5 , each of the degree measures of angles 1, 2, 3, 4, 8, 9, 10, and 11 equal 67.5° . Then, I observed that $\angle 5$ is congruent to $\angle 14$ because they are two complete angles of a regular octagon. Since each angle of a regular octagon equals 135° , then $\angle 5$ and $\angle 14$ both equal 135° .

On regular octagons, there are 4 pairs of parallel sides. The diagonal of the octagon connected two parallel endpoints on a pair of sides. This makes the diagonal of the octagon perpendicular to both sides. $\angle 6$ and $\angle 9$ are the angles formed by the perpendicular lines, so their degree measures are 90° . Angle 7 and $\angle 12$ are adjacent angles to angles 6 and 13, so in order to find their degree measure, I used the equation, $135 - 90$, $135 - 90 = 45^\circ$, so $\angle 7$ and $\angle 12$ both equal 45° .

Knowing that the degree measure of $\angle 11$ is 67.5° and the degree measure of $\angle 12$ is 45° , I can use the information to find the degree measure of $\angle 25$. Since these three angles ($\angle 11, \angle 12, \angle 25$) are the angles of one triangle, I used the equation, $180 - 67.5 - 45$, to find the measure of $\angle 25$.

I used this because any triangle's angle sum equals 180 and to find the degree measure of the third angle when knowing the other two angle's degree measures. The same strategy applies to finding the measure of $\angle 20$ because I knew the degree measure of $\angle 7$ and $\angle 8$ and those three angles formed a triangle. The measure of $\angle 7$ was 45° and $m\angle 8 = 67.5^\circ$, so I used also the expression $180 - 67.5 - 45$ to find the measure of $\angle 20$. $180 - 67.5 - 45 = 67.5^\circ$, so both angles 20 and 25 equal 67.5° .

$\angle 23$ is the vertical angle of $\angle 25$ and $\angle 22$ is the vertical angle of $\angle 20$, both the measure of $\angle 23$ and $\angle 22$ equals 67.5° (vertical angles are congruent).

$\angle 26$ and $\angle 24$ are the supplements of $\angle 25$, so their degree measure is found out by the expression $180 - 67.5$, $180 - 67.5 = 112.5$, so the degree measures of $\angle 26$ and $\angle 24$ are 112.5° . $\angle 19$ and $\angle 21$ are the supplements of $\angle 20$

and $\angle 20$ is congruent to $\angle 25$. The supplements of an angle (part of a congruent pair), are congruent to the supplements of the other angle in the congruent pair.

Therefore, $\angle 19$ and $\angle 21$ equal 112.5° . To find out the degree measure of $\angle 17$, I subtracted 67.5 and another 67.5 from 180. I did this because the $m\angle 23$ and $m\angle 22$ equal 67.5 and angles 17, 22, and 23 form a triangle. $180 - 2 \cdot 67.5$ equals 45° , so $\angle 17$ is 45° . $\angle 15$ is the vertical angle to angle 17, so $m\angle 15$ is 45° also. $\angle 18$ and $\angle 16$ are supplements to angle 17 so to find out their measure,

I subtracted 45° from 180° . $180 - 45 = 135^\circ$, so $m\angle 18$ and $m\angle 16$ both equal 135° . The final answer is:

- $\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4 \cong \angle 8 \cong \angle 9 \cong \angle 10 \cong \angle 11 \cong \angle 20 \cong \angle 22 \cong \angle 23 \cong \angle 25 = 67.5^\circ$;
 $\angle 7 \cong \angle 12 \cong \angle 15 \cong \angle 17 = 45^\circ$; $\angle 5 \cong \angle 14 \cong \angle 16 \cong \angle 18 = 135^\circ$; $\angle 19 \cong \angle 21 \cong \angle 24 \cong \angle 26 = 112.5^\circ$
 $\angle 6 \cong \angle 13 = 90^\circ$.