

TOOTHPICK TRIANGLES

Performance Standard 8A.G

Determine the number of toothpicks needed to build a figure in the pattern accordingly:

- *Mathematical knowledge:* Represent situations using variables.
- *Strategic knowledge:* Use variables or recursive techniques appropriately to investigate a pattern of shapes made with objects.
- *Explanation:* Explain completely and clearly what was done and why it was done.

Procedures

1. *In order to describe numerical relationships using variables and patterns (8A)*, students should experience sufficient learning opportunities to develop the following:
 - Investigate, describe and generalize a variety of patterns using variables or recursive techniques.
 - Represent situations using variables.
2. Provide each student a copy of the "Toothpick Triangles" task sheet and the rubric. Have students review and discuss the task to be completed and how the rubric will be used to evaluate it.
3. Have the students work individually to solve the problem. Do not help the students or guide their thinking as they solve the following problem:
 - A. The first four pictures in a sequence of figures made up of toothpicks are shown. If the sequence of figures continues with each figure increasing in size in a similar manner, describe the next figure in the sequence. Use a drawing to help, but include a written description of what the figure looks like, and how you know that your picture is correct.
 - B. How many toothpicks will be needed to create the 15th picture in the sequence? Explain your reasoning.
 - C. Write a formula that you could use to find the number of toothpicks needed to build the nth figure in the sequence, for any natural number n. Explain your reasoning.
4. Evaluate each student's work using the rubric and its guide to determine the performance level. Give each student a score in each of the three categories. A score of 4 should indicate completely correct solutions to all parts of the problem, with complete and correct justifications of their reasoning. A three should represent correct or nearly correct solutions to all parts, with only minor computational errors making their solutions inaccurate, their rationale should be sound, but may not be completely explained. A two would indicate that students have some idea about how to answer the questions, but make major errors in computation and/or reasoning that effects their answers. A one may have a correct answer for one part, but generally shows little understanding in their rationale for their procedures and processes. A score of zero generally reflects no correct responses and no logical rationale for their procedures and processes. Minor errors in computation include making errors in the actual addition or multiplication, rounding incorrectly. Major errors include using the wrong operations or formulas to relate terms. Evaluate each part as follows:
 - Part A: The students should show and provide a description of the next term of the sequence. It should be a figure that includes 5 triangles connected in a row, and needing 11 toothpicks to build.
 - Part B: The students should discuss the fact that it takes 2 extra toothpicks to create the next picture compared to the one before it. So it takes 31 toothpicks to build the 15th picture in the sequence. Student may also just start creating the triangles, recognizing that the number of triangles is the same as the number of the term, then just count the number of "toothpicks" used.
 - Part C: The students should find that the general formula for the number of toothpicks needed to create the figure is $2n+1$. When writing a formula, students should show proof and/or examples that justify their reasoning. Students should make sure all variables used are defined.Their explanations should tell how they found these answers and why the answers are correct.

Examples of Student Work follow

Time Requirements

- 20 - 25 minutes

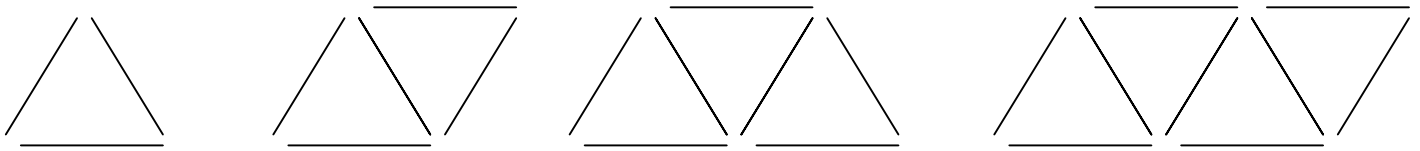
Resources

- Copies of the "Toothpick Triangles" task sheet
- Toothpicks may be supplied to help some students visualize and identify the pattern, but are not required.
- Mathematics Rubric

NAME _____ DATE _____

TOOTHPICK TRIANGLES

The first four pictures in a sequence of figures are shown. Each figure is made up of toothpicks. Assume that the sequence of figures continues with each figure increasing in size in a similar manner, and complete the following tasks:



- A. Describe the next figure in the sequence. You may use a drawing to help, but include a written description of what the figure looks like, and how you know that your picture is correct.
- B. How many toothpicks will be needed to create the 15th picture in the sequence? Explain your reasoning.
- C. Write a formula that you could use to find the number of toothpicks needed to build any figure in the sequence. Justify your formula and explain your reasoning.

Toothpick Triangles P

The first four pictures in a sequence of figures are shown. Each figure is made up of toothpicks. If the sequence of figures continues with each figure increasing in size in a similar manner, complete the following tasks.



- A. Describe the next figure in the sequence. You may use a drawing to help, but include a written description of what the figure looks like, and how you know that your picture is correct.



The figure has 2 more lines than the last figure. I know this because these lines make an additional triangle visible in the figure. Each new figure in the sequence has one more triangle than the last.

- B. How many toothpicks will be needed to create the 15th picture in the sequence? Explain your reasoning.



31 toothpicks are needed.
8 toothpicks on the bottom,
7 toothpicks on the top, and
16 toothpicks diagonal in the center.

This is 50 because in each figure, the number of center top and bottom together is the figure's sequence #, and the lines

- C. Write a formula that you could use to find the number of toothpicks needed to build the nth figure in the sequence, for any natural number n. Explain your reasoning.

$2n + 1 = \text{toothpicks}$

For example, the 1st figure $2 \times 1 + 1 = 3$ (toothpicks)

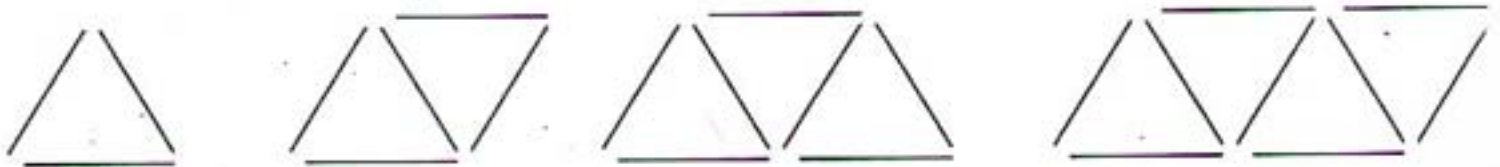
inside the figure is figure # + 1 (16)

4th Figure
 $2 \times 4 + 1 = 9$ toothpicks

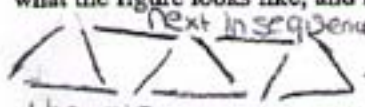
This pattern continues throughout the entire sequence.

Toothpick Triangles P

The first four pictures in a sequence of figures are shown. Each figure is made up of toothpicks. If the sequence of figures continues with each figure increasing in size in a similar manner, complete the following tasks.



- A. Describe the next figure in the sequence. You may use a drawing to help, but include a written description of what the figure looks like, and how you know that your picture is correct.

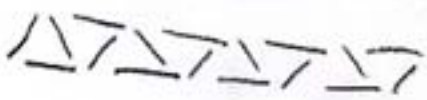


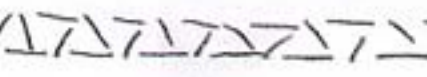
The next figure would look like that because the diagram have another triangle added to the end of the figure. The pattern keeps adding 2 toothpicks making the figure looks like a trapezoid when there is an odd number of triangles. It looks like a rhombus when there is an even number of triangles. I know my picture is correct because there is an odd number of triangles and it looks like a trapezoid.

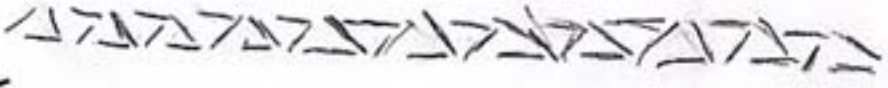
- B. How many toothpicks will be needed to create the 15th picture in the sequence? Explain your reasoning.
 It will take 31 toothpicks to create the 15th picture in the sequence. I took $14 \times 2 + 3$ because there were 3 toothpicks to start one triangle and then there was 14 triangles left and it only takes 2 toothpicks to create another triangle. so that's were the 14×2 takes it's part.

- C. Write a formula that you could use to find the number of toothpicks needed to build the nth figure in the sequence, for any natural number n. Explain your reasoning.

$2(n-1)+3$ because like I said before, there are three toothpicks to create the first triangle (3) and then for every other triangle you just take the number of triangles subtracted by one and multiply the number of triangles left by 2 (2n) and then add 3 to the product and you'll come up with how many toothpicks it takes to create that structure. ($2(n-1)+3 = \#$ of toothpicks)

ex.  $2(8-1)+3 = 17$ toothpicks

ex.  $2(15-1)+3 = 31$ toothpicks

 $2(21-1)+3 = 43$ toothpicks