

## ALTERNATIVE ALGORITHMS

### Performance Standard 6C.G

Analyze the appropriateness of the algorithms applied to six computations and determine if they will obtain the correct results accordingly:

- *Mathematical knowledge:* Use algorithms fluently for computing with rational numbers.
- *Strategic knowledge:* Determine the appropriateness of the algorithms and accuracy of the results.
- *Explanation:* Explain completely and clearly what was done and why it was done.

### Procedures

1. ***In order to compute and estimate using mental mathematics, paper-and-pencil methods, calculators, and computers (6C)***, students should experience sufficient learning opportunities to develop the following:
  - Analyze algorithms for computing with rational numbers and develop fluency in their use.
2. Provide each student with a copy of the "Alternative Algorithms" recording sheet and the rubric. Have students review and discuss the task to be completed and how the rubric will be used to evaluate it. Provide students with the necessary materials and resources.
3. Have the students work individually to solve the problems. (Do not help the students or guide their thinking as they solve the problem.)
4. Evaluate each student's work using the rubric and its guide to determine the performance level. The entire set of algorithms can be judged as a single item, using the following as a guide:
  - Minor errors in computation include making errors in the actual addition, subtraction, multiplication, or division that result in erroneous judgments. Major errors include not recognizing errors in the methods or not recognizing correct procedures for computation.
  - The students should find that the methods used in A, B, C & E lead to correct results, while D and F do not.
    - A.  $198 + 438 = 634$ ; uses a process of estimation and compensation.
    - B.  $346 - 158 = 188$ ; uses subtraction of each place value allowing negatives and then adding the results.
    - C.  $35 \times 16 = 560$ ; uses a mental math techniques of thinking of money, 35 cents is a quarter and a dime, so 16 sets of 35 cents would be 16 quarters and 16 dimes.
    - D.  $43 \times 16 = 688$  not 418; is a common error in mental math where students lose the 10 sets of 3 and the 6 sets of 40 when trying to use the distributive property mentally.
    - E.  $3654 \div 6 = 609$ ; uses the distributive property to divide 6 out of 3600 and 54, then add the results.
    - F.  $4875 \div 125 = 39$  not 95; overuses the distributive property and breaks down both the divisor and the dividend, which results in not dividing the entire dividend by the entire divisor.
  - Explanations need to state that "why" is not needed since the "what" is the goal of the exercise.

### Examples of Student Work follow

### Time Requirements

- 30 - 45 minutes

### Resources

- Piece of lined paper
- Copies of the "Alternative Algorithms" recording sheet
- Mathematics Rubric

NAME \_\_\_\_\_ DATE \_\_\_\_\_

## ALTERNATIVE ALGORITHMS

Students proposed the following computational methods when asked to do so without using the “typical” or standard algorithms. The original question asked of the students appears first, followed by their work. For each one, describe its method and decide if it will produce a correct result. If so, why? If not, why won’t it? (Record your responses on your lined paper.)

A.  $196 + 438$

$$\begin{aligned} 200 + 440 &= 640 \\ 640 - 4 &= 636 \\ 636 - 2 &= 634 \end{aligned}$$

B.  $346 - 158$

$$\begin{aligned} 300 - 100 &= 200 \\ 40 - 50 &= -10 \\ 6 - 8 &= -2 \\ 200 - 10 &= 190 \\ 190 - 2 &= 188 \end{aligned}$$

C.  $35 \times 16$

$$\begin{aligned} 35 &= 25 + 10 \\ 25 \times 16 &= 400 \\ 10 \times 16 &= 160 \\ 400 + 160 &= 560 \end{aligned}$$

D.  $43 \times 16$

$$\begin{aligned} 40 \times 10 &= 400 \\ 3 \times 6 &= 18 \\ 400 + 18 &= 418 \end{aligned}$$

E.  $3654 \div 6$

$$\begin{aligned} 3600 \div 6 &= 600 \\ 54 \div 6 &= 9 \\ 600 + 9 &= 609 \end{aligned}$$

F.  $4875 \div 125$

$$\begin{aligned} 4000 \div 100 &= 40 \\ 800 \div 20 &= 40 \\ 75 \div 5 &= 15 \\ 40 + 40 + 15 &= 95 \end{aligned}$$

4/2/01

Ⓐ The answer 634 is correct because they rounded to two numbers and the first number 196 was rounded up 4 numbers and the second number 438 was rounded up 2 numbers. So the student added  $200 + 440$  which equaled 640. Then subtract 4 from 640 because the number 196 was rounded up 4 numbers so the student got 636. Then the student subtracted 2 from 636 since the starting number, 438, was rounded up two numbers. So the student ended up with the correct answer which is 634.

Ⓑ The answer 188 is correct because the student rounded the number 346 down to 300 and the number 158 down to 100 and then subtracted the two to get 200. Then the student looked at about how much he/she rounded and it was 40-50 to equal -10. Then the student saw how much he/she rounded the last time and saw that it was 6-8 which would equal -2 and then added all of those numbers together...  $200 + -10 + -2 = 188$ .

Ⓒ The student divided the number 35 into  $25 + 10$ . Then the student multiplied  $25 \times 16$  to equal 400 and  $10 \times 16$  to equal 160. Then they added  $400 + 160$  to equal 560 which would be the correct answer.

① The student took  $40 \times 10 = 400$  and  $3 \times 6 = 18$  when the original numbers were 43 and 16. The way this student did the problem was wrong because he/she divided the ones from the tens and then multiplied them. Then added the answers together which would give them the answer of 418 which would be wrong because the right answer is 688.

② The student took the number 3654 and made it into two numbers:  $3600 + 54$ . Then the student took 3600 and divided it by 6 which would give them 600. Then they took 54 and divided it by 6 which would give them 9. Then they added  $600 + 9$  together to get the correct answer 609.

③ The student took  $4875 + 125$  and rounded them to  $4000 + 100$ . So then he/she divided  $4000 \div 100$  which would equal 40. Then the student rounded the number that was already rounded and came up with  $800 \div 20$  which would equal 40. Then the student took what was left which was  $75 \div 5$  which equaled 15. Then the student added  $40 + 40 + 15 = 95$ . The student came up with the WRONG answer and the correct answer is 39.

$$\begin{array}{r} 196 \\ + 438 \\ \hline 634 \end{array}$$

a. Yes, this problem is right. This is because 196 rounds to 200 & 438 rounds to 440. They added the numbers together & got 640. The difference between 196 & 200 is 4, so they subtracted that. They then took off the difference between 438 & 440, which is 2, so that gave them 634.

$$\begin{array}{r} 346 \\ - 158 \\ \hline 188 \end{array}$$

b. This problem is right. It is because the student broke the problem down. 346 stands for 300 + 40 + 6. 158 stands for 100 + 50 + 8. They took 300 - 100, giving them 200. They took 40 - 50, giving them -10. They also took 6 - 8, giving them -2. Then they added the numbers.  $200 + -10 = 190 + -2 = 188$ .

$$\begin{array}{r} 35 \\ \times 16 \\ \hline 210 \\ + 350 \\ \hline 560 \end{array}$$

c. This problem is also right. The student broke 35 down into 2 simpler numbers, 25 & 10. They multiplied 25 x 16, giving them 400. They multiplied 10 x 16, giving them 160. They added 400 + 160, giving them 560.

d. 43

$$\begin{array}{r} \times 16 \\ 258 \\ + 430 \\ \hline 688 \end{array}$$

This problem is wrong because they only multiplied  $40 \times 10 + 3 \times 6$ . They should have multiplied  $10 \times 40$  and  $10 \times 3$ . They also should have taken  $6 \times 40 + 6 \times 3$ . Then they add them all together.

e.

$$\begin{array}{r} 609 \\ 6 \overline{) 3654} \\ \underline{-36} \\ 54 \\ \underline{-54} \\ 0 \end{array}$$

This problem is right. They took  $3600 \div 6$ . Then they took  $54 \div 6$ . They added those answers to get 609.

f.

$$\begin{array}{r} 39 \\ 125 \overline{) 4875} \\ \underline{-375} \\ 1125 \\ \underline{-1125} \\ 0 \end{array}$$

This is wrong because they took  $4000 \div 100$ . They should have taken  $4000 \div 125$ . They did the same with the rest of the numbers.