MATCHING PAIRS

Performance Standard 10C.G

Determine the probability of various results of a game that is based on two-stage probability and determine the odds of winning the game accordingly:

- *Mathematical knowledge*: know the difference between odds and probability; know how to compute probabilities of simple compound events,
- *Strategic knowledge*: use appropriate strategies to determine the odds of winning the game, and
- *Explanation*: explain completely and clearly what was done and why it was done.

Procedures

1. Provide students with sufficient learning opportunities to develop the following skills in order to (10C) determine, describe and apply probabilities of events:
   - Discuss odds versus probability, and
   - Compute probabilities of simple compound events using such methods as organized lists and tree diagrams.
2. Provide each student a copy of the "Matching Pairs" task sheet and the rubric. Have students review and discuss the task to be completed and how the rubric will be used to evaluate it.
3. Have the students work individually to solve the problem. Do not help the students or guide their thinking. Students will also need to be familiar with a standard deck of playing cards. The game rules are:

   A carnival director has proposed the following game of chance, but needs to know the odds of winning. The player rolls a pair of dice and gets a sum between 2 and 12. The game operator then cuts a deck of cards and shows the bottom card from the cut. If the card cut is less than the sum of the dice then the player wins. If the sum is equal to the card's value, then the player "pushes" meaning they neither win nor lose, but they play again without paying. If the card is larger than the sum of the dice, then the player loses. Calculate the probability of winning and the probability of pushing. Discuss what this says about the odds of winning, and if the game is fair.

4. Evaluate each student's work using rubric and its guide to determine the performance level. Give each student a score in each of the three categories, scoring each part of the problem separately. The explanation for the game can be found on "Matching Pairs" resource sheet.

Examples of Student Work not available

Time Requirements
- One class period

Resources
- Copies of the “Matching Pairs” task sheet
- Teacher Resource Sheet
- Calculators should be as needed
- A pair of dice and a deck of playing cards to use in simulating the game may be helpful to some students
- Mathematics Rubric

MATCHING PAIRS TEACHER RESOURCE SHEET

The game is fairly simple to explain, but takes a little thought to work through to find the probabilities and odds of winning. The sample space looks like this:

Sum of Dice	**# of ways to get this sum**	**Cards for player win**	**Cards for player "push"**	**Cards where player loses**
2	1	4 aces	4 twos	44 (3-king)
3	2	8 (ace & 2)	4 threes	40 (4-K)
4	3	12 (1-3)	4 (4)	36 (5-K)
5	4	16	4	32
6	5	20	4	28
7	6	24	4	24
8	5	28	4	20
9	4	32	4	16
10	3	36	4	12
11	2	40	4	8
12	1	44	4	4
TOTAL	36	288	44	

Chances of winning need to take into account the chances of getting each roll. Since there are 36 possible combinations of the two dice, and 52 different cards that can be drawn the total number of outcomes is 36 x 13 which is 1872.

If you roll a two the chances of winning are only 4 our of 52, or 1 out of 13, but the chances of getting that two are only 1 out of 36.... so 1/36 times 4/52 is the same as 4 out of 1872 or 1 out of 468.

If you roll a three the chances of winning are only 8 out of 52, or 2 out of 13. The chances of rolling the three is only 2/36 or 1/18. So 2/36 times 8/52 is 16 out of 1872 or 1 out 117.

In a similar manner you can calculate the probability of all winning combinations. This is shown on the following table.

Winning combinations	Calculation	Probability of that combo
2 (A)	1/36 x 1/13	1/468
3 (A,2)	2/36 x 2/13	4/468 or 1/117
4 (a,2,3)	3/36 x 3/13	9/468 or 1/52
5 (a,2,3,4)	4/36 x 4/13	16/468 or 4/117
6 (a,2,3,4,5)	5/36 x 5/13	25/468
7 (a,2,3,4,5,6)	6/36 x 6/13	36/468 or 1/13
8 (a,2,3,4,5,6,7)	5/36 x 7/13	35/468
9 (a,2,3,4,5,6,7,8)	4/36 x 8/13	32/468
10 (a, 2,3,4,5,6,7,8,9)	3/36 x 9/13	27/468 or 3/52
11 (a,2,3,4,5,6,7,8,9,10)	2/36 x 10/13	20/468 or 5/117
12 (a,s2,3,4,5,6,7,8,9,10, J)	1/36 x 11/13	11/468
TOTAL		216/468

There are also 4 out of 52 or 1/13 chances of pushing for each of the 36 rolls, so that takes care of 4x36 or 144 additional outcomes out of the 1872 possible outcomes, or 1/13 or 36/468.

There are a total of 216 + 36 = 252 ways of winning or pushing out of 468, so the other 216 of the 468 must be ways to lose.

So the probability of winning is 216/468. The probability of losing is 216/468.

The odds of winning are 1:1.... meaning the game is fair, since you have an equal chance of winning and losing.

NAME ____ DATE ______________________________

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Student Task Sheet

A carnival director has proposed the following game of chance, but needs to know the odds of winning.

Game rules:
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Calculate the probability of winning and the probability of pushing.
Discuss what this says about the odds of winning, and if the game is fair.