

POLYGONS AND DIAGONALS

Performance Standard 9B.F

Find the measure of the interior angles of given regular polygons and create a formula for finding the measure of the interior angles of any polygon accordingly:

- *Mathematical knowledge:* know the measure of one angle of an equilateral triangle is 60 degrees. They will also know, therefore, the sum of the three angles is 180 degrees; know relationships between the number of vertices and/or sides a polygon has, the number of diagonals, and the sum of the interior angles of that polygon,
- *Strategic knowledge:* use prior knowledge about the sum of the interior angles of a triangle and appropriate strategies to find the correct measure of the interior angles in each of 10 regular polygons, and create a formula to find the measure of the interior angles of any regular polygon given the number of sides,
- *Explanation:* explain completely and clearly what was done and why it was done.

Procedures

1. ***In order to identify, describe, classify, and compare relationships using points, lines, planes and solids (9B)***, students should experience sufficient learning opportunities to develop the following:
 - Determine the relationships between the number of vertices or sides in a polygon, the number of diagonals, and the sum of the interior angles.
2. Provide each student a copy of the “Regular Polygons” and “Polygons and Diagonals” task sheets and the rubric. Have students review and discuss the task and how the rubric will be used to evaluate it.
3. Ask students to determine the measures of the interior angles of the 10 regular polygons on the task sheet (i.e., quadrilateral, pentagon, hexagon, heptagon, octagon, nonagon, decagon, 11-gon, dodecagon, 100-gon). For the student to do the majority of this assessment independently, the teacher needs to begin the task by discussing the triangle and then the square. The triangle has no diagonals because the sides connect at a point to form each vertex. The measures of the interior angles of a triangle when added together equal a sum of 180 degrees. Fill in the chart. Triangles have 3 sides, no diagonals, one triangle (itself), sum of the interior angles is 180 degrees and the measure of each regular angle would be 60 degrees. Now do the square. The student should use a straightedge to draw one diagonal. Only one is drawn because diagonals cannot cross each other. The student picks one vertex and connects it with all other vertices using diagonals. In the square when one vertex is chosen, there is only one other vertex (the one diagonal from it) that can be connected. The other two vertices are connected to the chosen vertex by the square’s sides. Fill in the chart: The number of sides is 4, diagonals is 1, number of triangles is 2, sum of angles is 360 degrees and the measure of each angle is 90 degrees. Multiply the number of triangles formed by 180 degrees to get the sum of the interior angles. Divide the sum by the number of vertices the polygon has to get the measure of each interior angle. With a straightedge, the student will draw the diagonals in all the other regular polygons provided, fill in the chart with the data s/her collects, and use the data collected to find the measure of the angles in each regular polygon.
4. In writing, have the student describe the pattern s/her sees in the collected data and then create a formula that can be used to find the measure of the interior angles of any polygon given (n) number of sides. (Formula: $(n-2) \times 180/n$. The formula reads: number of sides minus 2 times 180 divided by the number of sides.) The student will need to create this formula to give the measure of the angles of the last three polygons listed on the chart.
5. Evaluate each student’s work using the rubric and its guide to determine the performance level. Students should demonstrate knowing that the angles of an equilateral triangle are 60 degrees each and that the sum of the angles of a triangle, any triangle, is 180 degrees. They should also demonstrate knowing the angles of a square measure 90 degrees since each is a right angle. A diagonal would cut the right angle in half making two triangles with one 90-degree angle and two 45-degree angles. The sum of the measures of the three angles would be 180 degrees each. Their explanation would include the sum of the angles of the two triangles – 360 degrees – which would then be divided by 4 to give the measure of each angle – 90 degrees. This informal proof lays the groundwork for the rest of the polygons. By drawing diagonals, triangles are created. Multiply 180 by the number of triangles created. The product is the total number of degrees of all the angles. That total is then divided by the number of vertices and the quotient is the measure of each angle.

Examples of Student Work not available

Time Requirements

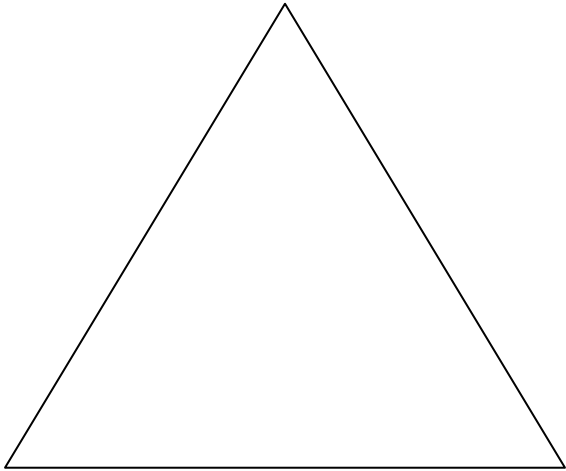
- One class period

Resources

- Copies of “Regular Polygons” (3 pages)
- Copies of the “Polygons and Diagonals” recording sheet
- Pencil, straightedge and calculator for each student
- Mathematics Rubric

REGULAR POLYGONS

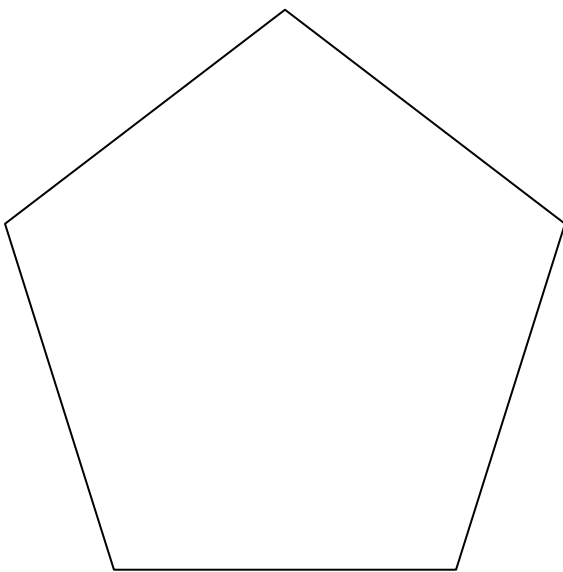
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TRIANGLE



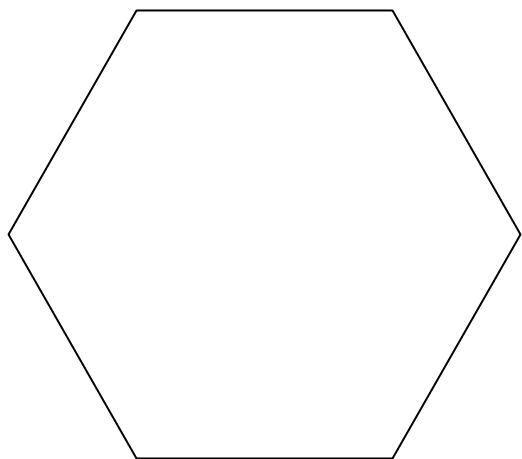
QUADRILATERAL



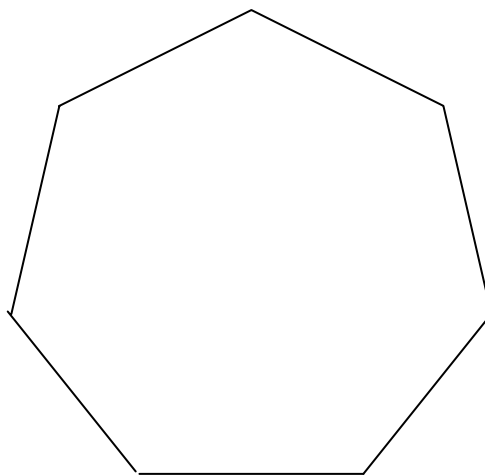
PENTAGON

REGULAR POLYGONS

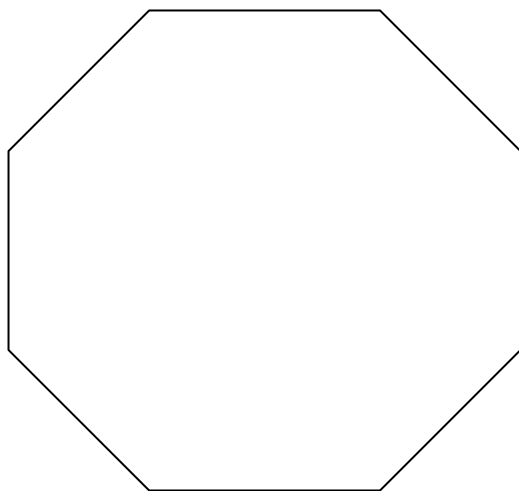
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HEXAGON



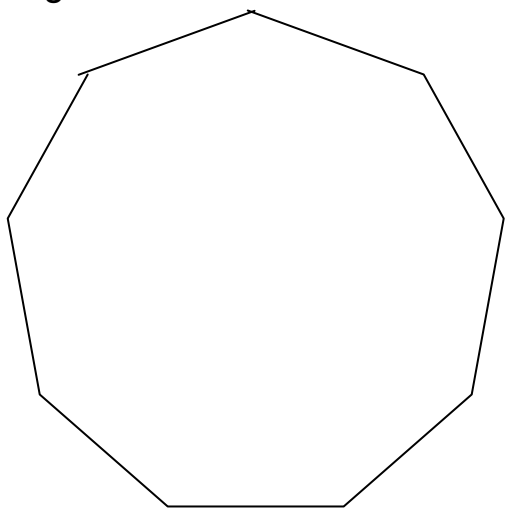
HEPTAGON



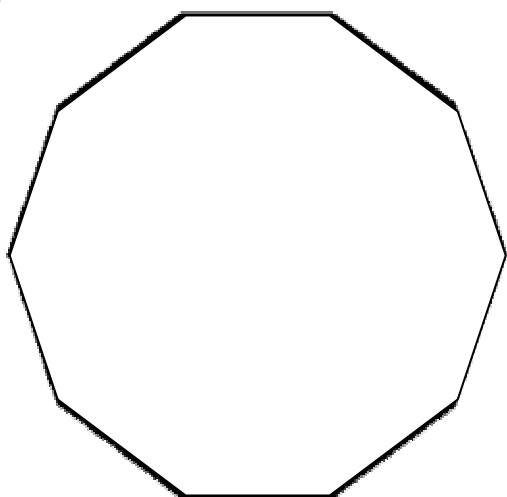
OCTAGON

REGULAR POLYGONS

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NONAGON



DECAGON

Name _____ Date _____

POLYGONS AND DIAGONALS

1. For each regular polygon, choose **one** vertex and, using a straightedge, draw **all** of the diagonals **from** that vertex.
2. Complete the table as far as the decagon.
3. In writing at the bottom of this sheet, describe the pattern you see.
4. Use the pattern you observed to create a formula for finding the measure of the interior angles of any regular polygon given (n), the number of sides. Use the formula to complete the last three rows on the chart.

Name of Shape	Number of sides	Number of diagonals	Number of triangles	Sum of <u>all</u> the angles	Measure of each regular angle
Triangle					
Quadrilateral					
Pentagon					
Hexagon					
Heptagon					
Octagon					
Nonagon					
Decagon					
11-gon					
Dodecagon					
100-gon					