PERIMETER CHAINS

Performance Standard (8A/8B/8C).E

Describe the pattern of change in the perimeters of chains of regular polygons using a table and a variable expression.

- *Mathematical knowledge:* Determine the perimeter of a chain of regular polygons and use this to write an expression with a variable for any number of blocks;
- *Strategic knowledge:* Use patterns to express the perimeter of a polygon chain with any number of polygons;
- *Explanation:* describe in writing how the perimeter of any regular polygon chain can be found using an equation and examples to support the conclusions

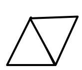
Procedures

1. ***In order to describe numerical relationships using variables and patterns (8A), interpret and describe numerical relationships using tables, graphs, and symbols (8B), and solve problems using systems of numbers and their properties (8C)***, students should experience sufficient learning opportunities to develop the following:
   - Describe, extend, and make generalizations about given geometric and numeric patterns.
   - Model problem situations with objects and equations to draw conclusions.
   - Solve problems with whole numbers using order of operations, equality properties, and appropriate field properties.
2. Each student should have 10 pattern block triangles, 10 pattern block squares, and 10 pattern block hexagons.
3. Begin with the triangles. The student needs to create a table on sheet of paper that looks similar to this:

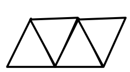
TRIANGLES

Number of triangles	1	2	3	4	5	6	7	8	9	10	…	n
Perimeter	3 units	4 units	5 units	6 units								

4. The student is to put one triangle on his/her desk. Ask what the perimeter of the triangle is and have the student write it in the box on the table.
5. Now have the student add one triangle. The second triangle should be pointing down so that one side touches a side of the first.



6. Ask the student what the perimeter of this figure is and put it on the chart under 2 triangles.
7. Have the student add another triangle, record the perimeter, add another and record the perimeter. (See chart above.)

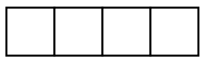


8. Ask the student if s/he is beginning to see a pattern. Ask him/her to describe the pattern in writing and tell why this pattern is occurring. Ask the student to create an expression to represent the pattern using the variable "n" as the number of triangles. (n+2) Now ask the student to create an equation to calculate the perimeter. (P=n+2) Have them use the equation to calculate the perimeter of 20 triangles in a chain. Working with a partner, the student can put 20 triangles together to prove the equation works.

9. Now have the student use the square pattern blocks to make a chain. S/he should create a table to record the number of squares and the perimeter as squares are added to discover the pattern.

SQUARES

Number of squares	1	2	3	4	5	6	7	8	9	10	…	n
Perimeter	4 units	6 units	8 units	10 units								

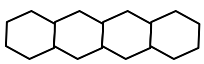


The expression to describe this pattern is 2n+2 and the equation is P=2n+2. Have the student describe and explain the pattern, expression, and equation in writing as before. The student should use the equation to calculate the perimeter of 20 squares and work with a partner to check his/her work.

10. Use the hexagons to create the next chain. The student should have a table to record number of hexagons and perimeter.

Hexagons

Number of hexagons	1	2	3	4	5	6	7	8	9	10	…	n
Perimeter	6 units	10 units	14 units	18 units								



The expression to describe this pattern is 4n+2 and the equation is P=4n+2. Have the student describe and explain the pattern, expression, and equation in writing as before. The student should use the equation to calculate the perimeter of 20 squares and work with a partner to check his/her work.

11. The student needs to describe and generalize in writing how the perimeter of any regular polygon chain could be found. S/he should use an equation and examples to support his/her conclusion.
12. When the student describes the pattern s/he should say something like "The number of units in the perimeter is always 2 more than the number of triangles used to create the chain. A triangle has three sides and two triangles would have six sides. When I put one triangle next to another with two sides touching, those two sides are no longer part of the perimeter. So instead of six units in the perimeter, there are only four. Each time a triangle is added, only two of the sides become part of the perimeter because the third side touches one side of the previous triangle. A way to express this using a variable is n for the number of triangles plus 2 for the two additional sides/units in the perimeter. P=n+2 means perimeter equals the number of triangles plus 2 units. "
13. When describing the pattern of the chain of squares, the student might write something like, "A square has 4 sides and a perimeter of 4 units. When another square is added the perimeter changes by only two units. The two sides that are next to each other are no longer part of the perimeter. The two top sides and the two bottom sides are part of the perimeter. So 2 squares contribute 2 units each to the perimeter plus the two end units that always remain. A way to express this is 2n+2. That means 2 units from the number of squares in the chain plus the two end sides/units."
14. When describing the pattern of the chain of hexagons, the student should now begin to see the pattern of only certain sides being part of the perimeter. Each new hexagon contributes four new units to the perimeter with the sum of the two ends being constant. It's important for the student to see this pattern of variable plus a constant in order to make that next step to describe the pattern of any regular polygon chain.
15. To make the next step, the student will see that the number of units contributed to the perimeter is two less than the number of sides it contains. These two sides disappear from the perimeter when the two polygons are placed together side by side. To express this using variables the student would write [(S-2) n] +2. "S-2" would stand for the number of sides of the polygon minus the two sides that disappear each time. In order of operations, this

part of the equation must be done first. That quantity is then multiplied times “n” which stands for the number of polygons in the chain. Add 2 to the product and you have the perimeter of any regular polygon chain.

Examples of Student Work follow

Time Requirements
- Two class periods

Resources
- Pattern blocks, specifically triangles, square and hexagons
- Pencil
- Paper
- Mathematics Rubric

Perimeter Chains

Directions

- *Mathematical knowledge:* determine the perimeter of a chain of regular polygons and use this to write an expression with a variable for any number of blocks
- *Strategic knowledge*: use patterns to express the perimeter of a polygon chain with any number of polygons
- *Explanation:* describe in writing how the perimeter of any regular polygon chain can be found using an equation and examples to support the conclusions

Materials

10 pattern block triangles
10 pattern block squares
10 pattern block hexagons
Math Rubric

Procedure

Triangle Chain

Step 1.

Number of Triangles	1	2	3	4	5	6	7	8	9	10	...	n
Perimeter	3	4	5	6	7	8	9	10	11	12		n+2

a. Put one triangle on your desk.
b. What is the perimeter? Record it on your table.
c. Add a 2nd triangle. (The second triangle should be pointing down so that one side touches a side of the first.)

d. What is the perimeter? ? Record it on your table under 2 triangles. Add another triangle, and record the perimeter; add another and record the perimeter.

e. Do you see a **pattern**? Describe it: yes, each time you add a triangle, the perimeter is the no of triangles plus two

f. Write an **expression** for this pattern. n+2

g. Write an **equation** to calculate the perimeter. P=n+2

h. Use the equation to calculate the perimeter of 20 triangles in a chain. Write answer here 22

i. Work with a partner and put 20 triangles together to prove the equation works.

j. Describe and explain the pattern, expression, and equation.

When you add a triangle, the perimeter equals the no of triangles plus 2 because when you push 2 triangles together, you lose one side from each triangle. Therefore instead of 6 sides you have 4

Squares

Number of Squares	1	2	3	4	5	6	7	8	9	10	...	n
Perimeter	4	6	8	10	12	14	16	18	20	22		n·2+n

Step 1

a. Make a chain using the square pattern blocks.

b. Record the number of squares and the perimeter as squares are added.

c. Do you see a **pattern**? Describe it: yes, each time you add a square the perimeter is the no of squares plus itself, plus 2

d. Write an **expression** for this pattern. $2n+2$

e. Write an **equation** to calculate the perimeter. $P=2n+2$

f. Use the equation to calculate the perimeter of 20 squares in a chain. Write answer here 42

g. Work with a partner and put 20 squares together to prove the equation works.

h. Describe and explain the pattern, expression, and equation.

i. When you add a square, the perimeter equals the no of squares times 2, and then plus 2 because when you push 2 squares together, you lose 1 side from each square. Therefore instead of 8 sides you have 6

Hexagons

Number of Hexagons	1	2	3	4	5	6	7	8	9	10	...	n
Perimeter	6	10	14	18	22	26	30	34	38	42		

Step 1

a. Make a chain using the hexagon pattern blocks.

b. Record the number of hexagons and the perimeter as hexagons are added.

c. Do you see a **pattern**? Describe it: yes, each times you add a hexagon the perimeter is the no. of hexagons times 4, and then plus 2

d. Write an **expression** for this pattern. $4n+2$

e. Write an **equation** to calculate the perimeter. $P=4n+2$

f. Use the equation to calculate the perimeter of 20 hexagons in a chain. Write answer here 82

g. Work with a partner and put 20 hexagons together to prove the equation works.

h. Describe and explain the pattern, expression, and equation.

When you add a hexagon the perimeter equals the no of hexagons times 4, and then plus 2. This is because when you push 2 hexagons together you lose 1 side from each hexagon so instead of 12 sides you have 10

"Exceeds" (page 5)

△ - n+2
□ - 2n+2
⬡ - 4n+2

Conclusion:

Explain how the perimeter of any regular polygon chain could be found. How would you express this in an equation?

$(n-2)n+2$ $\qquad$ n = number of sides

I did (n-2) because you lose the 2 sides that are touching. Next, I multiplied by n because after you subtract the 2 sides there are still the sides on the top/bottom (see ex). Next I added 2 because of the 2 on the sides (see ex).

ex. side  side
top
bottom